INVENTORY APPLICATIONS OF SERVOMECHANISM MODELS

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ABSTRACT

A dynamic inventory control model, formulated in terms of discrete variable servomechanism theory, is examined. The behavior of the system is first analyzed by digital computer simulation. Both random and deterministic inputs are considered. Two double echelon models are then constructed and examined analytically utilizing transform theory. Finally, the properties of forecasters of demand are analyzed for the cases where mean demand is constant and linear. Exponential smoothing techniques are compared with standard estimation procedures.

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1. INTRODUCTION

This paper will take for its starting point a dynamic inventory control model formulated in servomechanism theory by Vassian [6] and extended by Reilly [5] and Zehna [7]. The techniques of linear system analysis as developed by Howard in [2] and the exponential smoothing/forecasting techniques of Brown [1] are heavily relied upon. The model as it is described in [7] will henceforth be referred to as the "basic model." Briefly, in the basic model we have demand as the input and inventory level, measured relative to some safety level, as the output. Inventory is replenished using a reorder rule which is based on a linear combination of past demand and inventory level feedback. Units of inventory are assumed to be issued instantaneously and a fixed lead time is assumed for receipt of replenishment orders. In discrete variable servo theory notation the two equations which describe the basic model are

(1-1) $\quad I_t = I_{t-1} + \theta_{t-1-T} - X_t$ and

(1-2) $\quad \theta_t = \sum_{j=o}^{t} A_j X_{t-j} + \sum_{j=o}^{t} B_j I_{t-j}$

where $I_t \equiv$ on hand inventory relative to some safety level at time t.

$\theta_t \equiv$ reorder quantity computed at time t.

$\{A_j\} \equiv$ a set of constants which weight past demands.

$\{B_j\}$ ≡ a set of constants which weight past inventory levels.

Let $D(z) \equiv \sum_{k=o}^{\infty} d_k z^k$, be the z-transform of the sequence $\{d_k\}$.

Then, the above equations become, in transform notation,

(1-3) $I(z) = zI(z) + z^{T+1} \theta(z) - X(z)$ and

(1-4) $\theta(z) = A(z)X(z) + B(z)I(z)$.

It is easily shown that

(1-5) $$I(z) = \frac{[z^{T+1} A(z) - 1]}{1-z - z^{T+1} B(z)} \cdot X(z)$$

Let

(1-6) $$S(z) = \frac{[z^{T+1} A(z) - 1]}{1-z - z^{T+1} B(z)}$$

be called the transfer function. Then (1-5) may be written

$$I(z) = S(z)X(z).$$

Similarly,

(1-7) $\theta(z) = [A(z) + B(z)\ S(z)]\ X(z)$

$= S'(z)\ X(z)$

where $S'(z) \equiv [A(z) + B(z)\ S(z)]$

the basic model is summarized in flow graph form in Figure 1.

As Vassian suggests [6], we will let

(1-8) $$B(z) = \frac{-(1-z)}{1-z^{T+1}}$$

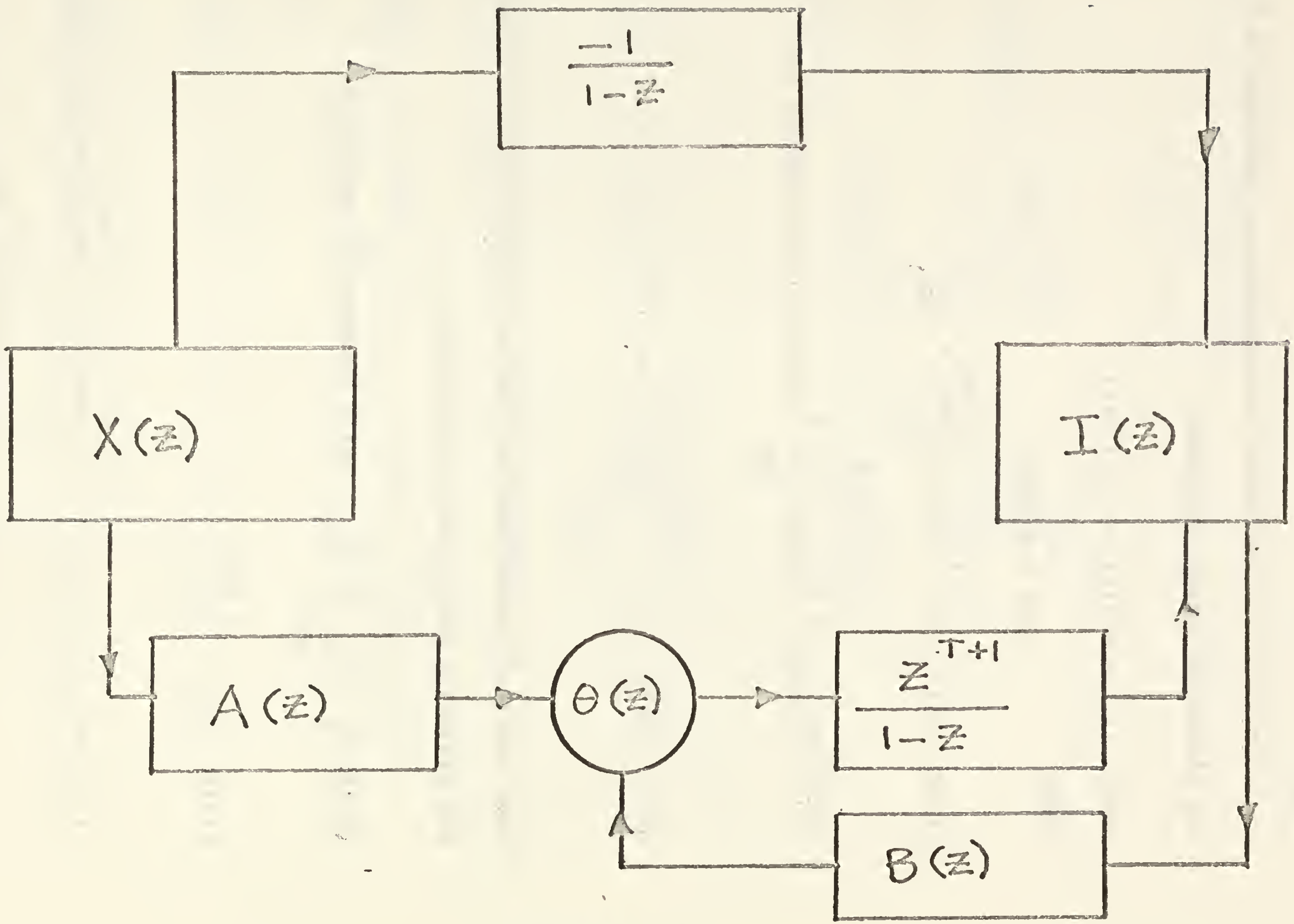

FIGURE 1

FLOW GRAPH DIAGRAM OF THE BASIC MODEL

It follows then that S(z) will have no finite poles. In that case, the denominator of S(z) will be finite for all finite values of z.

The following additional relationships, developed in [7], are listed for ready reference:

(1-9) $E(z) \equiv E[I(z)] = S(z)\, m(z)$

where

$$m(z) = E[X(z)]$$

(1-10) $A^*(z) = z^{T+1}\, A(z)\, \dfrac{(1-z^{T+1})}{1-z}\, X(z)$

(1-11) $A^*_{t+1+T} = I_{t+1+T} + \sum_{j=o}^{T} X_{t+1+T-j}$. We require that

(1-12) $E[A^*_{t+1+T}] - \sum_{j=t+1}^{t+1+T} m(j) \to 0$ as $t \to \infty$

When expected demand does not vary with time ($E[X_t] = a$) we have the following additional relationships.

(1-13) $\hat{a}_t = \alpha \sum_{k=o}^{t} \beta^k X_{t-k}$ using exponential smoothing.

(1-14) $\hat{a}(z) = \dfrac{\alpha X(z)}{1 - \beta z}$

(1-15) $A^*_{t+1+T} = (T+1)\, \hat{a}_t$, a forecast of demand during lead time plus one unit of time.

$$(1\text{-}16)\quad A^*(z) = \frac{\alpha\,(T+1)\, z^{T+1}\, X(z)}{1 - \beta z}$$

$$(1\text{-}17)\quad A(z) = \frac{\alpha\,(T+1)\,(1-z)}{(1-z^{T+1})(1-\beta z)}$$

$$(1\text{-}18)\quad S(z) = \frac{\alpha\,(T+1)\, z^{T+1}}{1-\beta z} - \frac{(1-z^{T+1})}{1-z}$$

$$(1\text{-}19)\quad I(z) = \frac{\alpha\,(T+1)\, z^{T+1}\, X(z)}{1-\beta z} - \frac{(1-z^{T+1})\, X(z)}{1-z}$$

$$(1\text{-}20)\quad \theta(z) = S'(z)\, X(z) = \left[\frac{\alpha\,(T+1)\,(1-z)}{1-z} + 1\right] X(z)$$

Inverting the transform in (1-20), we obtain

$$(1\text{-}21)\quad \theta_t = \alpha\,(T+L) \sum_{k=o}^{t} \beta^k X_{t-k} - \alpha\,(T+L) \sum_{k=o}^{t-1} \beta^k X_{t-1-k} + X_t$$

and

$$(1\text{-}22)\quad \theta_t = \alpha\,(T+1)\,[X_t - \hat{a}_{t-1}] + X_t\,.$$

Inverting the transform in (1-19) yields

$$(1\text{-}23)\quad I_t = \alpha\,(T+1) \sum_{k=o}^{t-T-1} \beta^k X_{t-T-1-k} - \sum_{k=o}^{T} X_{t-k}$$

Computing the expected value of I_t we obtain

$$(1\text{-}24)\quad E_t = \alpha\,(T+1) \sum_{k=o}^{t-T-1} \beta^k\, m(t) - \sum_{k=o}^{T} m(t)$$

$$= \alpha\,(T+1)\, a \sum_{k=o}^{t-T-1} \beta^k - (T+1)\, a$$

$$= \alpha\,(T+1)\, a \,\frac{[\,\beta^{t-T} - 1]}{\beta - 1} - (T+1)\, a \quad \text{or,}$$

$$(1\text{-}25)\quad E_t = -a\,(T+1)\,\beta^{t-T}$$

Thus, we have described the basic model and displayed the resulting mathematical relationships. In addition, for the case of constant mean demand we have derived explicit formulae for the expected on hand inventory and reorder quantities at time t when exponential smoothing is used to estimate the mean.

2. DIGITAL COMPUTER SIMULATION

The simulation presented here was programmed in FORTRAN 60 and run on the CDC 1604 serial number 1. The programs used are included in Appendix II in the order of discussion below.

First, a deterministic simulation of the basic model with constant mean was made based on the relation

$$E_t = \alpha(T+1) \sum_{k=o}^{t-T-1} \beta^k \, m(t) - \sum_{k=o}^{T} m(t)$$

letting $\alpha = .2$, $\beta = .8$, and $T = 4$. In the first run $m(t)=16$ and the results of 995 iterations are shown in Appendix I-A. It is seen, that to six decimal place accuracy, the expected inventory level reaches zero after 83 time periods. This is merely a simple verification of the analytical results shown in [7].

In the second deterministic run, we let $m(t)=t$ in the basic relation. Since Reilly [5] has shown that first degree exponential smoothing yields biased results when the demand is linear we would expect this bias to be reflected in the expected inventory. In the case at hand we have

$$E_t = \alpha(T+1) \sum_{k=o}^{t-T-1} \beta^k \, m(t-T-1-k) - \sum_{k=o}^{T} m(t-k)$$

Letting $\alpha = .2$, $\beta = .8$, $m(t) = t$ and $T = 4$,

$$E_t = \sum_{k=o}^{t-5} (.8)^k \, (t-5-k) - \sum_{k=o}^{4} (t-k)$$

$$E_t = t\sum_{k=o}^{t-5} (.8)^k - 5\sum_{k=o}^{t-5} (.8)^k - \sum_{k=o}^{t-5} k(.8)^k - \sum_{k=o}^{4} t + \sum_{k=o}^{4} k$$

$$= t\left[\frac{1-(.8)^{t-4}}{.2}\right] - 5\left[\frac{1-(.8)^{t-4}}{.2}\right]$$

$$+ .8\left[\frac{-(1-.8^{t-4}) - .2(t-4)\ .8^{t-5}}{.04}\right] -5t + 10$$

$$= -5t(.8)^{t-4} + 45(.8)^{t-4} - 4t(.8)^{t-5} + 16(.8)^{t-5} - 35.$$

Hence, $\lim_{t \to \infty} E_t = -35$

We can see from the results shown in Appendix I-B that in fact $E_t = -35$ after 78 iterations again to six decimal place accuracy. So we have verified the analytical conclusions in [5] and [7], i.e., that the model is asymptotically unbiased (no steady state error in the expected inventory level) whenever asymptotically unbiased forecasters are used and asymptotically biased when correspondingly biased forecasters are used. However, as a periodic review inventory model, the basic model may require an impractical number of periods to converge to an acceptable level. Conceding that convergence can be accelerated by adjusting α and T, we must face the fact that asymptotic unbiasedness may not be a practical measure of effectiveness for an inventory model.

The second simulation to be considered is stochastic in nature. Assume that demand is Poisson, i.e.,

$$P[X_t = k] = \frac{e^{-\lambda_t} \lambda_t^k}{k!} \quad \text{where } \lambda_t = \lambda .$$

For ease of simulation a normal random number generator was employed taking advantage of the fact that for large λ [3]

$$\frac{e^{-\lambda} \lambda^k}{k!} \doteq \frac{1}{\sqrt{2\pi}} \int_{\frac{k-\lambda - .5}{\sqrt{\lambda}}}^{\frac{k-\lambda + .5}{\sqrt{\lambda}}} e^{-\frac{1}{2}y^2} \, dy.$$

The 1000 random numbers for the first two runs are listed in Appendix III-A, while those used in runs three and four are shown in Appendices III-B and III-C respectively.

In the first run $\lambda_t = 16$ in the basic relation

$$I_t = \alpha(T+1) \sum_{k=o}^{t-T-1} \beta^k X_{t-T-1-k} - \sum_{k=o}^{T} X_{t-k} .$$

I_6 thru I_{1000} are listed from right to left in Appendix I-C. It is seen that there is some tendency toward stability through I_{15} but after that time period we observe the effects of the random demand on the inventory level. It is now clear that carrying expected values through the model does not adequately describe the true situation.

In the second stochastic run, all parameters and variables were unchanged from the first run except that T was increased to T = 10. We see the resulting inventory values in Appendix I-D.

As expected, some increase in the variation of I_t is observed, i.e., the variance of the random variable I_t is a function of the lead time of the basic model. However, we also observe that I_t is relatively insensitive to T in the sense that the order of magnitude of the change in T is not carried over to I_t to an appreciable degree.

In the third stochastic run, all parameters and variables were the same as in the first run except that $\lambda_t = 100$. Naturally, to preserve our Normal approximation, the Poisson distribution variance was also increased to 100. Note that the random numbers generated and listed in Appendix III-B reflect this change. Now we observe the resulting values of I_t shown in Appendix I-E. Analogously we see that the variance of I_t depends on the variance of X_t but is relatively insensitive. The order of magnitude of the increase in λ is not reflected in I_t.

In the final run λ_t was allowed to vary with time, specifically $\lambda_t = 16 + t$. From the analytical development we are led to expect bias since our forecast is biased for non-constant mean demand. In Appendix III-C observe the new set of random numbers generated with mean $\lambda_t = 16 + t$ and variance $\lambda_t = 16 + t$. The expected bias is apparent from an inspection of the Appendix I-F. Most values of I_t are negative. Again observe the relative insensitivity of I_t to increasing the variance of the demand.

In summary, the computer simulations have confirmed the theoretical results as expected. However, in doing so, some

limitations of the basic model have been suggested. Asymptotic unbiasedness as a criterion appears to be weak and probably impractical even when expected values of the random variables X_t and I_t are considered. When $\{X_t\}$ is a sequence of realizations of a random variable from a probability distribution, the sequence $\{I_t\}$ certainly does not appear to approach zero, Vassian's [6] apparent claim to the contrary. Zehna [7] clarified the theory in this area but the results of the simulation dramatically emphasize the inadequacy of the basic model when only expectations are considered. The choice of a safety level, or other managerial decisions, cannot be made on the basis of $E[I_t]$ alone but $Var[I_t]$ must be accounted for also.

3. DOUBLE ECHELON MODELS

It is possible to formulate multiple echelon models in servo theory along the lines suggested by Howard [2]. It is desired to test the basic model of the previous sections in echelon. First, we construct a double echelon model using three basic models, two at the retail level and one at the wholesale supporting stock point position. Assume that consumer demand with constant mean values a_1 and a_2 is the system input at the retail level. In addition to the inventory level outputs at the retail stock points we have the reorder quantities $\theta_1(t)$ and $\theta_2(t)$ serving as demand inputs to the wholesale stock point. There is a lag time which is constant for each retail stock point of T_1 and T_2 respectively between the time of issue by the wholesale stock point and receipt by the retail points. Assume that the wholesale stock point has a sufficiently high safety level so that it is always able to fill orders from the retail activities. There are to be no interactions between the retail stock points. All three stock points are to use a single exponential smoothing forecaster which is of course asymptotically unbiased for constant mean demand. The lead time, T_3, for the wholesale stock point is the delivery time from the manufacturer. Assume that all three stock points use the same value of the smoothing constant α. Hence this double echelon model describes, in a simple way, a common situation involving two echelons of inventory. The use of a common inventory control doctrine and the assumption of no interactions at the retail level are considered

to be reasonable in the light of the author's experience. It is then of interest to determine the possible effects of such an overall doctrine in terms of this model.

In the framework of the multiple input-output system in [4] we have a system with two inputs $X_1(z)$, $X_2(z)$ and three outputs $I_1(z)$, $I_2(z)$ and $I_3(z)$. In matrix notation this becomes

$$[I_1(z), I_2(z), I_3(z)] = [X_1(z), X_2(z)] \begin{bmatrix} h^T_{11}(z) & 0 & h^T_{13}(z) \\ 0 & h^T_{22}(z) & h^T_{23}(z) \end{bmatrix}$$

where, for the present case,

$$h^T_{11}(z) = S_1(z) = \frac{z^{T_1+1}\ \alpha(T_1+1)}{1-\beta z} - \frac{1-z^{T_1+1}}{1-z}$$

and $$h^T_{22}(z) = S_2(z) = \frac{z^{T_2+1}\ \alpha(T_2+1)}{1-\beta z} - \frac{1-z^{T_2+1}}{1-z} \;.$$

Hence, $I_1(z) = S_1(z)\ X_1(z)$ and

$$I_2(z) = S_2(z)\ X_2(z) \;.$$

This double echelon model is graphed in Figure 2. Now

$$I_3(z) = S_3(z)\ [\theta_1(z) + \theta_2(z)]$$

since the input at the wholesale stocking point is

$$\theta_1(z) + \theta_2(z).$$

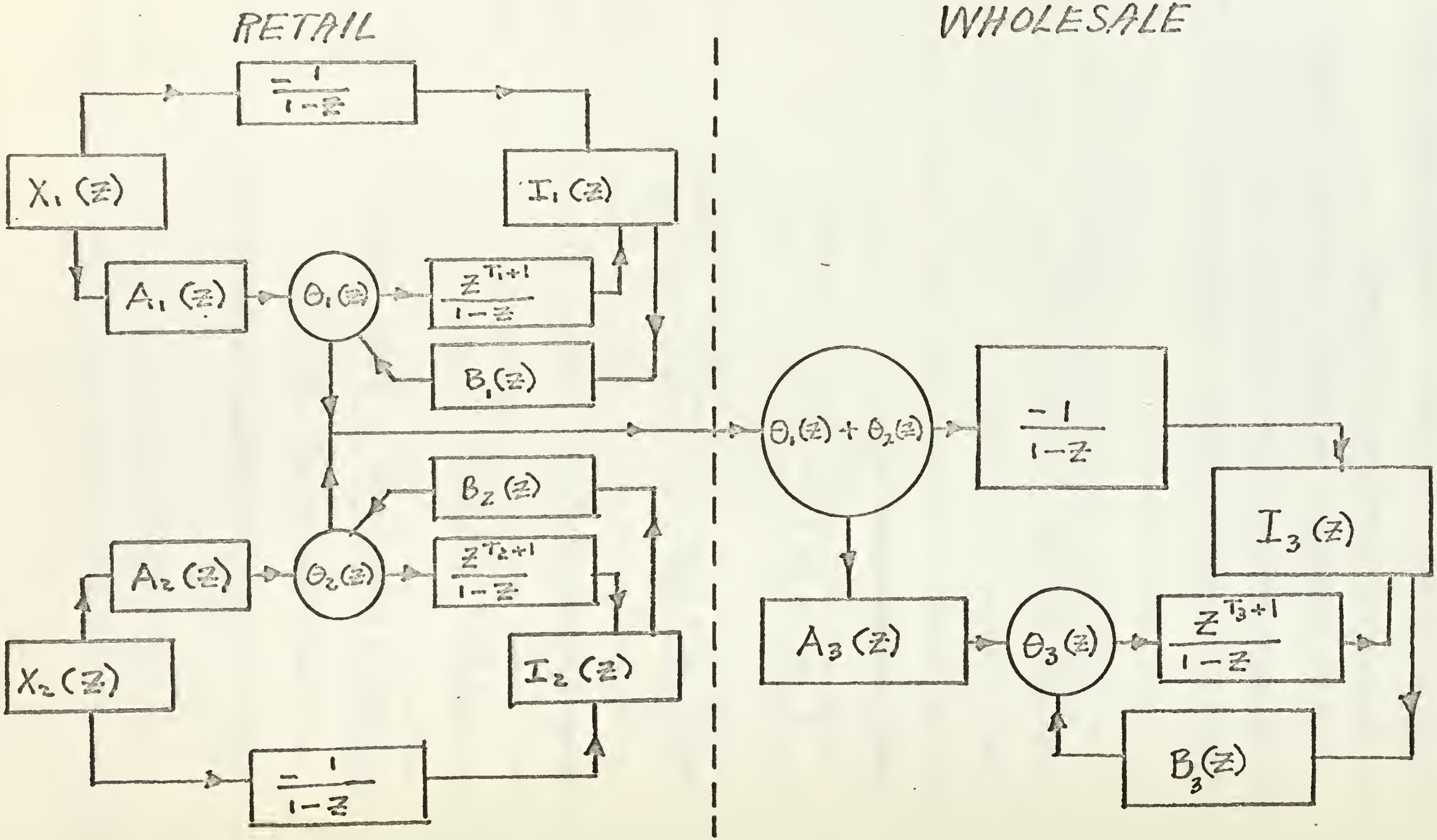

FIGURE 2

FLOW GRAPH DIAGRAM OF RETAIL/WHOLESALE DOUBLE ECHELON MODEL

From (1-7) we have

$$I_3(z) = S_3(z)\ \{[A_1(z) + B_1(z)\ S_1(z)]\ X_1(z) + [A_2(z)+B_2(z)S_2(z)] \cdot X_2(z)\}$$

Hence,

$$h^T_{13}(z) = S_3(z)\ [A_1(z) + B_1(z)\ S_1(z)] \quad \text{and}$$

$$h^T_{23}(z) = S_3(z)\ [A_2(z) + B_2(z)\ S_2(z)]\ .$$

Recall that we are assuming

$$m_1(t) = E[X_1(t)] = a_1 \text{ and } m_2(t) = E[X_2(t)] = a_2 \text{ for } t \geq 0.$$

Hence,

$$m_1(z) = \frac{a_1}{1-z} \quad \text{and } m_2(z) = \frac{a_2}{1-z} \quad .$$

Consider now

$$\theta_i(t) = \alpha\ (T_i+1) \sum_{k=o}^{t} \beta^k\ X_{t-k} - \alpha\ (T_i+1) \sum_{k=o}^{t-1} \beta^k\ X_{t-1-k} + X(t)$$

so that

$$\theta_i(z) = \left[\frac{\alpha(T_i+1)\ (1-z)}{1 - \beta z} + 1 \right] \cdot X_i(z)$$

for $i = 1, 2$.

$$E[\theta_i(t)] = \alpha(T_i+1)\ a_i \sum_{k=o}^{t} \beta^k - \alpha(T_i+1)\ a_i \sum_{k=o}^{t-1} \beta^k + a_i$$

$$E[\theta_i(t)] = \alpha(T_i+1)\ a_i \frac{(\beta^t-1)}{\beta-1} - \alpha(T_i+1)\ a_i \frac{(\beta^{t-1}-1)}{\beta-1} + a_i$$

$$= a_i[1-(T_i+1)\ (\beta^t-1) + (T_i+1)\ (\beta^{t-1}-1)]$$

$$= a_i[1-(T_i+1)\ \beta^t + (T_i + 1)\ \beta^{t-1}\ ]$$

$$= a_i[1+(T_i+1)\ (\beta^{t-1} - \beta^t)]$$

$$= a_i[1+(T_i+1)\ \beta^{t-1}\ (1 - \beta)]$$

$$= a_i[1+(T_i+1)\ \alpha\ \beta^{t-1}\ ]\ .$$

This demonstrates that the expected demand at the wholesale stock point, which is the sum of the order quantities from the retail stock points $[\theta_1(t) + \theta_2(t)]$, is not a constant despite the constant mean demand inputs, $m_1(t)$ and $m_2(t)$, to the system as a whole. It is clear then that the inventory control doctrine at one echelon does affect the stability of the system. In this case, a constant mean demand is transmitted as an exponential function because of the exponential smoothing forecasting system employed.

Now it is of interest to examine the behavior of the wholesale inventory level under these conditions. Consider

$$S_3(z)\,[A_1(z) + B_1(z)\,S_1(z)]\,X_1(z)$$

which is the first component of $I_3(z)$ shown above. By assumption

$$S_3(z) = \left[\frac{\alpha(T_3+1)\,z^{T_3+1}}{1-\beta z} - \frac{1-z^{T_3+1}}{1-z}\right].$$

$$[A_1(z) + B_1(z)\,S_1(z)]\,X_1(z)$$

$$= \left[\frac{\alpha(T_1+1)\,(1-z)}{(1-z^{T_1+1})(1-\beta z)} - \frac{z^{T_1+1}\,\alpha(T_1+1)(1-z)}{(1-z^{T_1+1})\,(1-\beta z)} + 1\right] \cdot \frac{a_1}{1-z}.$$

$$= \left[\frac{\alpha(T_1+1)\,(1-z) + (1-\beta z)}{1-\beta z}\right] \cdot \frac{a_1}{1-z}$$

Hence,

$$S_3(z)[A_1(z) + B_1(z)\,S_1(z)]\,X_1(z) = S_3(z)\,S_1'(z)\,X_1(z)$$

$$= \left[\frac{\alpha(T_3+1)\,z^{T_3+1}}{1-\beta z} - \frac{1-z^{T_3+1}}{1-z}\right]\left[\frac{\alpha(T_1+1)(1-z)+(1-\beta z)}{1-\beta z}\right] \cdot \frac{a_1}{1-z}.$$

Now,

$$\lim_{z \to 1} \left[\frac{\alpha(T_3+1) z^{T_3+1}}{1 - \beta z} - \frac{1-z^{T_3+1}}{1 - z} \right] = 0 \quad \text{from [7, p. 15],}$$

and obviously

$$\lim_{z \to 1} \frac{\alpha(T_1+1)\ (1-z) + (1-\beta z)}{1 - \beta z} = 1 \ .$$

Therefore,

$$\lim_{z \to 1} (1-z)\ S_3(z) = 0$$

and since

$$E_3(z) = S_3(z)\ [S_1'(z)\ X_1(z) + S_2'(z)\ X_2(z)] \ ,$$

it follows that

$$\lim_{z \to 1} (1-z)\ E_3(z) = 0 \ .$$

Based on a well known theorem [4, p. 218], the result above implies that $E_t \to 0$ as $t \to \infty$.

At first this may seem to be unexpected until one realizes that asymptotic unbiasedness is not a necessary condition for the steady state error to vanish. Recall that, in transfer notation,

$$(1-z) E_3(z) = S_3(z) S_1'(z)\ a_1 + S_3(z) S_2'(z)\ a_2 \ .$$

Thus with constant mean demand, the most that is required for asymptotic unbiasedness is that $\lim_{z \to 1} S_3(z) = 0$ and that $\lim_{z \to 1} S_1'(z)$ be a constant. Since $S_3(z)$ depends only on A(z)

and $B(z)$, it is relatively easy to choose $A(z)$ and $B(z)$ to yield the limit zero. $S_1'(z)$ here is a linear combination of the asymptotic unbiased retail level transfer function and hence will be a constant in the limit under these conditions. Therefore, the results above are indeed natural under the circumstances. If initial expected demand is not a constant ($m(z) \neq a_1/(1-z)$ then $S_3(z) \rightarrow 0$ and $S_i'(z) \rightarrow c$ for some constant c may not be sufficient conditions for no steady state error. Again this emphasizes the weakness of asymptotic unbiasedness as a criterion.

Inventory managers have a keen interest in the number of units in the pipeline (units issued from one stock point but not reflected on the records of another). Let us see what this double echelon model reveals about the problem. Remember it was assumed that issues are made instantaneously by the wholesale stock point but T_1+1 and T_2+1 time periods elapse before the inventory is recorded at the respective retail points. We are interested in the expected number of units in the double echelon system which are not shown on the records of any stock point as inventory.

The total number of units on order at time t by a retail stock point can be written

$$Q_t = \sum_{j=1}^{T} Q_{t-j} = \sum_{j=1}^{T} \alpha(T + 1) \sum_{k=o}^{t-j} \beta^k X_{t-j-k}$$

Hence,

$$E[Q_t] = \alpha\, (T + 1) \sum_{j=1}^{T} \sum_{k=o}^{t-j} \beta^k a$$

$$E[Q_t] = \alpha(T+1)\ a \sum_{j=1}^{T} \sum_{k=o}^{t-j} \beta^k = \alpha(T+1)\ a \sum_{j=1}^{T} \frac{(\beta^{t-j+1}-1)}{\beta - 1}$$

$$= -a(T+1) \sum_{j=1}^{T}(\beta^{t-j+1}-1) = aT(T+1) - a(T+1) \sum_{j=1}^{T} \beta^{t-j+1}$$

$$= aT(T+1) - a(T+1) \frac{(\beta^{t+1}-1)}{\beta - 1} + a(T+1) \frac{(\beta^{t-T+2}-1)}{\beta - 1}$$

$$= aT(T+1) + a \frac{(T+1)}{\alpha} \left[\beta^{t+1} - \beta^{t-T+2} \right]$$

$$= aT(T+1) + \frac{a(T+1)\ \beta^{t-T+2}}{\alpha} (\beta^{T-1} - 1)$$

$$= aT(T+1) - \frac{a(T+1)\ \beta^{t-T+2}(1-\beta^{T-1})}{\alpha} .$$

Then, $\lim_{t \to \infty} E[Q_t] = aT(T+1)$

for any retail point with $E[X_t] \equiv a$

and so

$$\lim_{t \to \infty} E[Q_t] = a_1 T_1 (T_1 + 1) + a_2 T_2 (T_2 + 1)$$

for the present double echelon model.

This is a natural result and the steady state condition is a direct consequence once again of the asymptotically unbiased forecasting. Note that the expected amount in the pipeline is a function of the square of lead time which re-emphasizes the possible savings resulting from a reduction in the lead time in any real situation. Then it appears that models of this type are useful for further study of pipeline inventory.

An inventory control point (ICP) is an office whose function is to manage a so-called commodity area which usually consists of a large number of stock items. A substantial portion of its effort involves the maintenance of optimal inventory levels at a system of stock points. The stock points report changes in their inventory levels and forward replenishment orders to the ICP periodically. The ICP takes centralized procurement action. One point of great interest is the correlation between inventory at the ICP and the inventory as it really exists at the stock points. It is possible for incompatible inventory control doctrines at the various stock points as well as different lead times, to generate a record of demand and inventory at the ICP which differs greatly from the actual demand input to the system and the inventory at the stock points. A very simple double echelon model which operates on the basis given above can be analyzed.

Assume there are two stock points operated as basic models with constant mean demand. Again the mean demands a_1 and a_2 are not necessarily the same nor are the lead times T_1 and T_2. There

are no interactions between the stock points. The reorder quantities generated at the two stock points, $\theta_1(t)$ and $\theta_2(t)$, are transmitted to an inventory control point whose operation differs from that of the basic model. It is assumed that, unlike the stock points, the control point has no physical inventory. It merely keeps a record of system inventory carried at the stock points based on the inputs it receives. Besides the two order quantities, the ICP receives a report of the inventory level at each stock point at each time period. Material ordered by the ICP is delivered direct to the stock points so that the lead time T_3 is a part of T_1 and T_2. The control point does not feed back its recorded inventory level but utilizes the values of $I_1(t)$ and $I_2(t)$ to determine order quantities $\theta_3(t)$ and update $I_3(t)$. The basic equations which describe the operation of the ICP are therefore

$$I_3(t) = I_1(t-1) + I_2(t-1) + \theta_3(t-1-T_3) - [\theta_1(t) + \theta_2(t)]$$

$$\theta_3(t) = \sum_{j=o}^{t} A_j\, [\theta_1(t-j) + \theta_2(t-j)] + \sum_{j=o}^{t} B_j\, [I_1(t-j) + I_2(t-j)]$$

and in transform notation

$$I_3(z) = z^{T_3+1}\, \theta_3(z) - \theta_1(z) - \theta_2(z) + z\, [I_1(z) + I_2(z)]$$

$$\theta_3(z) = A_3(z)\, [\theta_1(z) + \theta_2(z)] + B_3(z)\, [I_1(z) + I_2(z)] \quad .$$

The flow graph for this double echelon model is shown in Figure 3. The inventory control point's system is graphed in Figure 4.

Now let us examine the behavior of the ICP inventory record in relation to the actual inventories.

$$I_3(z) = z\,[I_1(z)+I_2(z)] + z^{T_3+1}\Big\{A_3(z)[\theta_1(z)+\theta_2(z)]$$

$$+ B(z)[I_1(z)+I_2(z)]\Big\} - [\theta_1(z)+\theta_2(z)]$$

$$= z[S_1(z)X_1(z) + S_2(z)X_2(z)] + z^{T_3+1}\Big\{A_3(z)[S_1'(z)X_1(z)+S_2'(z)]$$

$$+ B(z)[S_1(z)X_1(z) + S_2(z)X_2(z)]\Big\} - [S_1'(z)X_1(z)+S_2'(z)X_2(z)]$$

$$= [zS_1(z)+z^{T_3+1} A_3(z)S_1'(z) + B_3(z)S_1(z)-S_1'(z)]\,X_1(z)$$

$$+ [zS_2(z) + z^{T_3+1} A_3(z)S_2'(z) + B(z)S_2(z)-S_2'(z)]\,X_2(z)$$

$$= S_{13}(z)X_1(z) + S_{23}(z)X_2(z)$$

where

$$S_{13}(z) = zS_1(z) + z^{T_3+1} A_3(z)S_1'(z) + B(z)S_1(z) - S_1'(z) \quad \text{and}$$

$$S_{23}(z) = zS_2(z) + z^{T_3+1} A_3(z)S_2'(z) + B(z)S_2(z) - S_2'(z) \quad .$$

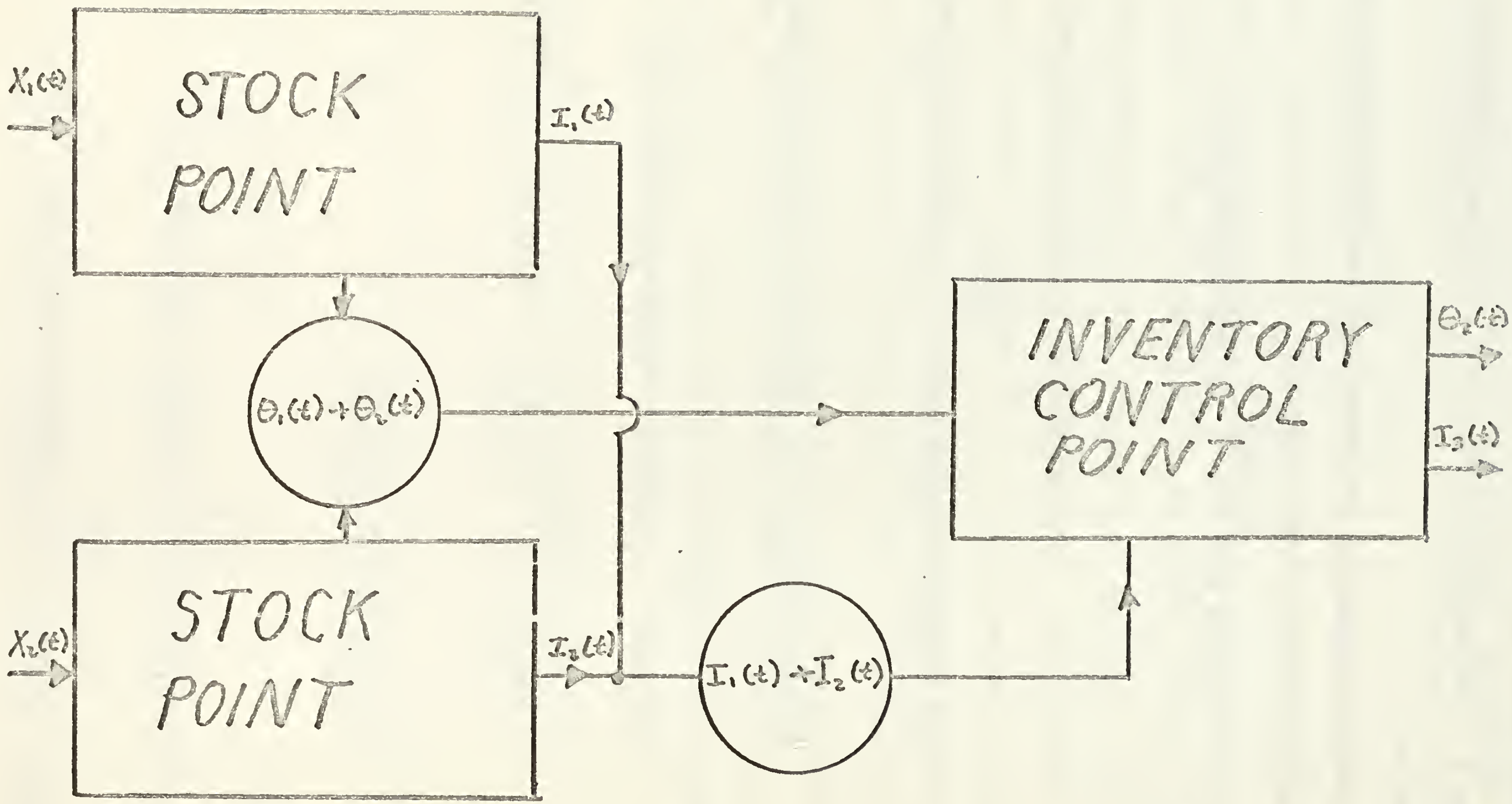

FIGURE 3
BLOCK DIAGRAM OF STOCK POINT/INVENTORY CONTROL POINT DOUBLE ECHELON MODEL

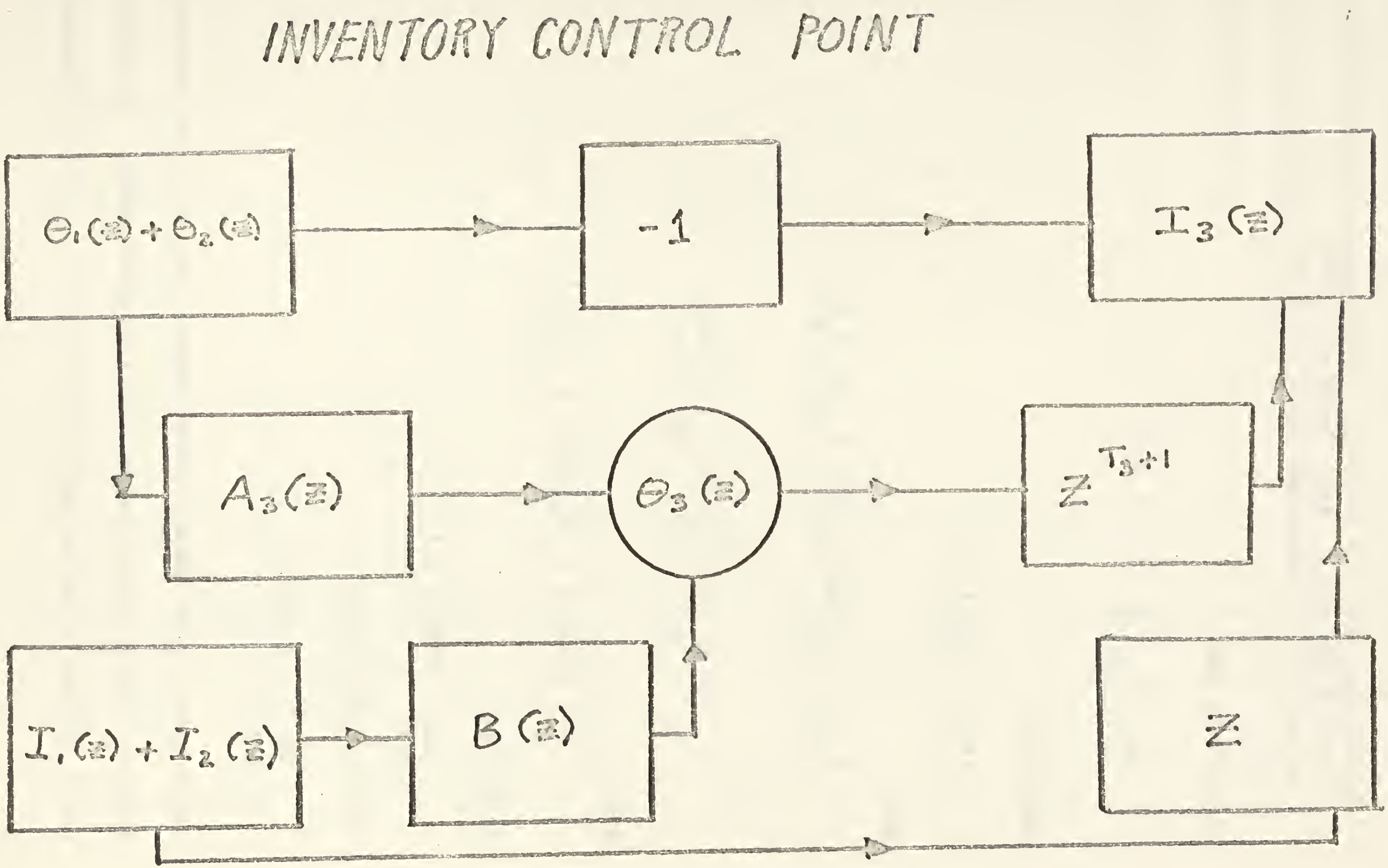

FIGURE 4

FLOW GRAPH DIAGRAM OF THE INVENTORY CONTROL POINT

Since

$$\lim_{z \to 1} S_1(z) = 0, \quad \lim_{z \to 1} A_3(z) = 1,$$

$$\lim_{z \to 1} S_1'(z) = 1, \quad \lim_{z \to 1} B(z) = -\frac{1}{T_3+1},$$

it follows that

$$\lim_{z \to 1} S_{13}(z) = 1 - 1 = 0.$$

Similarly,

$$\lim_{z \to 1} S_{23}(z) = 0.$$

But,

$$E_3(z) = S_{13}(z)\, m_1(z) + S_{23}(z)\, m_2(z)$$

$$= S_{13}(z) \frac{a_1}{1-z} + S_{23}(z) \frac{a_2}{1-z}$$

and

$$(1-z)\, E_3(z) = a_1\, S_{13}(z) + a_2\, S_{23}(z).$$

Therefore,

$$\lim_{z \to 1} (1-z)\, E_3(z) = 0,$$

which implies that

$$\lim_{t \to \infty} E_3(t) = 0.$$

Again there is no steady state error. Basically the results here parallel those in the first double echelon model. In effect, major difficulties can exist in the type of system discussed

above but the "no steady state error" criterion does not really measure the behavior of the output function adequately. It appears feasible, however, to formulate multiple echelon servo-mechanism inventory models which include ICP type operations in order to gain further insight into the operation of such a complex system.

4. DEMAND FORECASTING

Reilly [5] and Zehna [7] apply exponential smoothing to define forecastors in the cases where mean demand is constant and also when it is linear. Moreover, in terms of the basic model presented here, Zehna [7] has demonstrated that the expected inventory level converges to zero whenever any asymptotically unbiased forecastor is used. For example, in the constant and linear models, if least squares estimators are used to estimate the parameters and these are substituted in the forecastor, then, being unbiased, the forecastor is, a fortiori, asymptotically unbiased and the inventory level will converge to zero. The justification, then for using exponential smoothing lies in the advantages which relate to digital computer storage and mathematical tractability in the servomechanism model. See Brown [1] for a complete treatment of the advantages of exponential smoothing. However, these advantages are external to the present requirement of unbiasedness in the limit.

Under the model $m(t) = bt$, Zehna [7], following Reilly [5], uses $\hat{b}_t = \frac{\alpha}{\beta}[S_t(X) - S_t^{[2]}(X)]$ where $S_t^{[2]}(X)$ represents the sequence X_t doubly exponentially smoothed. It follows that $\hat{b}_t$ is an asymptotically unbiased estimator for b so that the forecaster

$$A^*_{t+1+T} = \sum_{j=t+1}^{t+T+1} \hat{b}_t j$$

is an asymptotically unbiased estimator for total demand over

the next lead time. For this model, another possibility suggests itself quite naturally. Since,

$$E[S_t(x)] = S_t(\xi) \doteq bt - \frac{b\beta}{\alpha}$$

$$E[S_t(x) - x(t)] \doteq - \frac{b\beta}{\alpha'}$$

and

$$\lim_{t \to \infty} E[\frac{\alpha}{\beta}(X(t) - S_t(x))] = b$$

Hence, the estimator

$$\hat{b}_t = \frac{\alpha}{\beta}[X(t) - S_t(x)]$$

is also asymptotically unbiased for the parameter b. Using this to define a forecaster

$$A^*_{t+T+1} = \sum_{j=t+1}^{t+T+1} \hat{b}_t j \qquad \text{it follows that}$$

$$E[A^*_{t+T+1}] = E\left[\sum_{j=t+1}^{t+T+1} \hat{b}_t j\right] \doteq \sum_{j=t+1}^{t+T+1} bj = \sum_{j=t+1}^{t+T+1} m(j)$$

so that A^*_{t+1+T} defines an asymptotically unbiased estimator for demand. Again by Zehna [7], $\lim_{t \to \infty} E_t = 0$ when this forecaster is used. Since this basic result is the same as that obtained when double exponential smoothing is used, other means will have to be used for comparing the two.

First consider,

$$E\left\{\frac{\alpha}{\beta}[S_t(x) - S_t^{[2]}(x)]\right\} = \frac{\alpha}{\beta}[S_t(\xi) - S_t^{[2]}(\xi)]$$

$$E\left\{\frac{\alpha}{\beta}[S_t(x)-S_t^{[2]}(x)]\right\} = \frac{\alpha}{\beta}\left\{\xi_t(1-\beta^t)-\frac{b\beta}{\alpha}[1-\beta^t(1-t)-t\beta^{t-1}]\right.$$

$$\left.-\xi_t(1-\beta^t)+\frac{b\beta}{\alpha}[1-\beta^t(1-t)-t\beta^{t-1}]+\frac{b\beta}{\alpha}[1-\beta^t]-\frac{t(t+1)b\,\alpha\beta^t}{2}\right\}$$

$$(4\text{-}1) \qquad = \frac{\alpha}{\beta}\left\{\frac{b\beta}{\alpha}[1-\beta^t]-\frac{t(t+1)b\,\alpha\beta^t}{2}\right\}$$

$$= \frac{\alpha}{\beta}\left\{\frac{b\beta}{\alpha}-\frac{b\beta^{t+1}}{\alpha}-\frac{t(t+1)b\,\alpha\beta^t}{2}\right\}$$

$$= b - b\beta^t - \frac{t(t+1)b\,\alpha^2\beta^t}{2\,\beta} \quad .$$

On the other hand

$$E\left\{\frac{\alpha}{\beta}[X(t)-S_t(x)]\right\} = \frac{\alpha}{\beta}[bt - S_t(\xi)]$$

$$(4\text{-}2) \qquad = \frac{\alpha}{\beta}\left\{bt-\xi_t(1-\beta^t)+\frac{b\beta}{\alpha}[1-\beta^t(1-t)-t\beta^{t-1}]\right\}$$

$$= \frac{\alpha}{\beta}\left\{bt-bt+bt\beta^t+\frac{b\beta}{\alpha}[1-\beta^t(1-t)-t\beta^{t-1}]\right\}$$

$$= \frac{\alpha}{\beta}\left\{bt\beta^t+\frac{b\beta}{\alpha}[1-\beta^t+t\beta^t-t\beta^{t-1}]\right\}$$

$$= \frac{\alpha}{\beta}\left[\frac{b\beta}{\alpha}(1-\beta^t)+\frac{b\beta}{\alpha}(t\beta^t-t\beta^{t-1})+bt\beta^t\right]$$

$$E\left\{\frac{\alpha}{\beta}\left[X(t) - S_t(x)\right]\right\} = \frac{\alpha}{\beta}\left[\frac{b\beta}{\alpha}(1-\beta^t) + \frac{bt\beta^t}{\text{[illegible]}}(\beta-1) + bt\beta^t\right]$$

$$= \frac{\alpha}{\beta}\left[\frac{b\beta}{\alpha}(1-\beta^t)\right] = \frac{\alpha}{\beta}\left[\frac{b\beta}{\alpha} - \frac{b\beta^{t+1}}{\alpha}\right]$$

$$= b - b\beta^t .$$

Obviously as $t \to \infty$ (4-1) converges to b more rapidly than (4-2) does and so is preferable.

5. CONCLUSIONS AND RECOMMENDATIONS

The main conclusions reached here are that expected inventory levels and asymptotic unbiasedness appear to be rather weak criteria for the models considered in this paper. It would be highly desirable to be able to relate costs and effectiveness to these models. The inventory variance should be the basis for any discussion of effectiveness (expected percentage of demands issued) and the quality of the estimator. Average cost depends on inventory carrying costs, order costs and shortage costs mainly. It is important that the decision criteria be related to these costs.

Many possible courses of action involving the ramifications of the basic model exist. It is recommended that the following two extensions be attempted first to determine whether or not this type of model has sufficient power to be useful:

(1) Calculate the $\mathrm{Var}[I_t]$ for several types of random demand and forecasting schemes. Explore the methods of determining a safety level to provide a given degree of protection against shortages.

(2) Attempt to define a new criterion for acceptable inventory level behavior which can be related to the commonly considered costs discussed above. Some insight into the problem might be gained by relating the lack of steady state error criterion to costs. Specifically, cost would seem to depend on the amount the inventory level varies from the norm, the relative

values of the cost components, the number of times I_t changes direction and the rate of convergence of E_t.

For this model to be useful it is necessary that the reorder policy be optimal in the sense of minimum cost. After a minimum cost criterion is applied to the model, the stochastic process $\{I_t, t = 0,1,2,\cdots\}$ must be intimately understood.

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APPENDIX I-A

MEAN VALUES

-51.200000	-40.960000	-32.768000	-26.214400	-20.971520
-16.777216	-13.421773	-10.737418	-8.589935	-6.871948
-5.497558	-4.398047	-3.518437	-2.814750	-2.251800
-1.801440	-1.441152	-1.152922	-.922337	-.737870
-.590296	-.472237	-.377789	-.302231	-.241785
-.193428	-.154743	-.123794	-.099035	-.079228
-.063383	-.050706	-.040565	-.032452	-.025961
-.020769	-.016615	-.013292	-.010634	-.008507
-.006806	-.005445	-.004356	-.003484	-.002788
-.002230	-.001784	-.001427	-.001142	-.000913
-.000731	-.000585	-.000468	-.000374	-.000299
-.000239	-.000192	-.000153	-.000123	-.000098
-.000078	-.000063	-.000050	-.000040	-.000032
-.000026	-.000021	-.000016	-.000013	-.000011
-.000008	-.000007	-.000005	-.000004	-.000003
-.000003	-.000002	-.000002	-.000001	-.000001
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-.000000	-.000000	-.000000	-.000000	-.000000

APPENDIX I-B

MEAN VALUES

-19.000000	-22.200000	-24.760000	-26.808000	-28.446400
-29.757120	-30.805696	-31.644557	-32.315645	-32.852516
-33.282013	-33.625610	-33.900488	-34.120391	-34.296313
-34.437050	-34.549640	-34.639712	-34.711770	-34.769416
-34.815533	-34.852426	-34.881941	-34.905553	-34.924442
-34.939554	-34.951643	-34.961314	-34.969052	-34.975241
-34.980193	-34.984154	-34.987324	-34.989859	-34.991887
-34.993510	-34.994808	-34.995846	-34.996677	-34.997342
-34.997873	-34.998299	-34.998639	-34.998911	-34.999129
-34.999303	-34.999442	-34.999554	-34.999643	-34.999715
-34.999772	-34.999817	-34.999854	-34.999883	-34.999906
-34.999925	-34.999940	-34.999952	-34.999962	-34.999969
-34.999975	-34.999980	-34.999984	-34.999987	-34.999990
-34.999992	-34.999994	-34.999995	-34.999996	-34.999997
-34.999997	-34.999998	-34.999998	-34.999999	-34.999999
-34.999999	-34.999999	-34.999999	-35.000000	-35.000000
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-35.000000	-35.000000	-35.000000	-35.000000	-35.000000
-35.000000	-35.000000	-35.000000	-35.000000	-35.000000
-35.000000	-35.000000	-35.000000	-35.000000	-35.000000

APPENDIX I-C

INVENTORY ON HAND

-40.000000	-42.800000	-32.840000	-17.872000	-29.897600
-40.718080	-29.574464	-28.459571	-36.167657	-23.934126
-7.147300	-5.717840	2.025728	12.220582	11.576466
4.461173	7.568938	5.655150	5.324120	-1.740704
-4.192563	11.045950	6.636760	-5.090592	-9.472474
-21.177979	-12.942383	-11.953907	8.236875	18.589500
18.671600	2.737280	-8.410176	-19.728141	-15.582513
-8.266010	-9.412808	2.469753	15.775803	15.020642
6.016514	11.013211	-1.989431	2.408455	13.126764
4.701411	-6.838871	-10.471097	-24.176877	-26.341502
-2.873202	5.301439	12.041151	-.767079	3.186337
-7.450931	-10.360745	-19.48896	-3.590877	1.927299
7.141839	-.286529	17.570777	12.856622	9.085297
6.868238	3.294590	-.364328	-.691462	-4.553170
12.557464	.245971	-5.003223	-9.402578	-1.922063
-12.337650	-3.070120	12.543904	-.164877	5.268099
15.614479	11.291583	11.833266	11.866613	4.293291
2.634632	-6.492294	-20.193835	-14.155068	-13.724055
-21.179244	-9.943395	9.245284	23.396227	8.116982
-1.306415	-6.445132	-21.956105	-27.564884	-6.651907
5.278474	-1.177221	-14.141777	5.886579	-1.690737
-3.352590	6.717928	15.374343	6.499474	5.799579
21.239663	10.791731	13.633385	11.906708	12.325366
3.660293	11.128234	3.502587	8.002070	10.001656
12.601325	-1.518940	1.584848	4.067878	14.254303
-3.196558	-6.757246	-25.005797	-23.804638	-33.843710
-15.274968	4.180026	6.744020	3.995216	7.796173
6.436938	5.349551	5.679641	6.743712	1.194970
-7.044024	-14.235219	-4.988175	-14.390540	-10.912432
1.870054	-2.903957	-5.923165	-7.933532	-8.950826
-.360661	17.911471	-.270823	2.183342	8.346673
-8.322661	-14.658129	-8.326503	-10.861203	-1.488962
3.408830	-1.472936	3.021651	3.417321	-12.666143
-7.532914	4.973668	2.178935	13.143148	20.114518
17.291615	.633292	-9.893367	-9.714693	-6.571755
-3.057404	-9.245923	1.203262	.962609	11.970087
23.576070	19.660856	8.928685	-2.857052	-12.285642
-21.428513	-11.742811	2.805751	24.644601	16.715681
22.772545	20.018036	3.414429	-6.468457	-10.774766
-17.419813	-7.935850	-3.348680	.921056	10.336845
-5.530524	-6.024419	-8.019535	2.384372	4.907497
13.725998	8.980798	-.615361	-8.692289	-1.353831
-11.083065	-16.266452	-.413162	9.269471	8.215577
15.772461	1.617969	-14.105625	-24.684500	-11.947600
-11.558080	-13.246464	.802829	8.042263	4.233810
1.987048	13.989639	-1.608289	6.313369	.450695
-1.239444	-8.991555	-4.593244	3.325405	18.260324
23.208259	11.766607	8.213286	-6.229371	-10.983497
-8.986798	-.389438	9.088449	10.870760	7.096608
4.077286	3.861829	-5.710537	-15.768430	-20.814744
-7.451795	-6.361436	2.910851	-4.671319	-11.937055
-14.949644	-.359715	-4.087772	4.529782	11.223826
6.179061	2.143248	-4.285401	-5.828321	-6.462657
-9.370125	-10.496100	10.603120	17.082496	31.265997
26.012797	19.010238	8.408190	6.926552	-12.258758
-22.007007	-12.205605	-19.764484	-24.611587	-15.289270
-12.031416	-13.225133	3.419894	.335915	5.268732
-2.385014	1.091989	2.273591	10.618873	-8.704902
2.836078	4.868863	3.095090	-.123928	2.900858
9.520686	.616549	-1.506761	6.994591	8.195673
8.156538	11.325231	7.060185	-6.351852	.318518
1.454815	6.763852	-5.388919	-3.911135	-8.128908
-12.903126	-22.122501	-7.298001	4.161599	1.729279
-5.416576	8.866739	.693391	-5.645287	-9.316230
-6.052984	3.557613	6.246090	8.796872	12.437498
14.349998	-11.120001	-9.296001	-14.436801	-.149441
7.080447	18.664358	5.531486	-.774811	-10.019849
-9.215879	-15.372703	-10.898163	6.081470	1.865176

-.907859	-2.126287	16.898970	7.119176	12.695341
11.156273	19.125018	23.300015	19.640012	12.312009
9.649607	-5.680314	-12.344251	-16.475401	-17.380321
-19.904257	-25.523405	-22.018724	-2.614979	7.708016
4.166413	6.533131	-1.373496	-1.098796	-1.679037
13.056770	11.645416	.116333	.293066	-4.165547
-11.132438	-8.505950	-1.804760	-4.243808	-2.795046
-4.236037	.211170	12.568936	3.255149	-3.595881
7.123295	18.298636	7.238909	-7.608873	-8.687098
-15.149679	-5.519743	-1.415794	13.467365	23.773892
8.419113	-3.664709	-13.531767	-12.025414	-20.820331
-8.256265	-6.805012	-.444010	-1.155208	9.275834
7.020667	8.616534	18.493227	16.594582	16.875665
3.900532	.920426	5.536341	9.229072	-10.616742
-2.293394	-8.034715	-24.227772	-21.382218	-.105774
.315381	-2.347695	13.121844	15.697475	4.357980
5.086384	3.669107	-11.064714	-16.451771	-19.561417
-15.249134	-6.199307	-8.159446	-6.327556	13.337955
11.670364	6.936291	16.349033	16.079226	2.063381
-5.549295	7.960564	-.631549	-9.905239	-8.124191
-23.699353	-27.559482	-15.847586	2.521931	7.217545
32.974036	24.379229	24.903383	22.722706	17.178165
-.457468	6.034026	1.027221	-.978224	-.782579
-5.426063	-3.740850	5.207320	-2.034144	-6.227315
-9.981852	-15.985482	-17.988386	-1.590708	-3.672567
6.461947	-6.430443	16.855646	17.884517	18.707613
16.566091	19.452873	-7.037702	-23.230162	-16.184129
-11.547303	-8.837843	3.329726	9.463781	-2.828975
1.136820	8.509456	.607565	-3.713948	-4.771159
-4.016927	-13.613542	-13.290833	-13.632667	-11.306133
-14.444907	-20.955925	-19.364740	-15.691792	-14.353434
5.917253	22.133802	35.707042	30.365634	35.692507
18.554005	2.643204	-2.685437	8.051651	-7.958679
-10.566943	-8.053555	.957156	-6.034275	.772580
1.018064	-5.785549	-12.828439	-13.862751	-20.690201
-16.152161	3.278271	19.822617	14.258094	16.406475
4.325180	8.460144	-6.031885	-3.625508	-1.500406
9.199675	-11.440260	10.447792	5.958234	10.566587
2.853269	11.682616	-8.853908	2.516874	-6.186501
-18.749201	-11.999361	-7.399488	-11.319591	4.144327
5.115462	9.492370	9.393896	2.515116	-.187907
-4.550325	-.040260	2.167792	11.534233	7.627387
.901909	-2.878473	-19.302778	-10.042222	-5.633778
-16.907022	-18.725618	4.419506	-4.254395	.788484
3.630787	.304630	-15.956296	-9.765037	-8.012030
11.390376	16.912301	18.929841	10.743873	-3.604902
-8.083922	-5.467137	-.173710	6.661032	6.128826
9.303061	9.842448	9.473959	-.620833	-18.096666
-7.477333	-12.781866	-15.025493	-6.020395	3.983684
-3.213053	-.570442	-3.656354	-9.325083	-7.860066
-13.688053	-5.350442	-9.880354	-2.104283	-7.083427
2.933259	3.346607	12.077286	13.661828	28.529463
16.423570	7.338856	-8.128915	-6.303132	-14.042506
-3.234005	11.412796	22.130237	3.304190	11.043352
14.434681	-1.052255	-11.441804	.846557	-5.122755
-13.298204	-3.838563	2.329150	.863320	-2.309344
8.952525	17.962020	21.569616	23.255693	22.204554
9.363643	-2.709085	-9.167268	-10.333815	-19.267052
-22.613641	-15.490913	-10.792730	-20.634184	-.907348
-6.525878	5.779298	-1.776562	6.978750	9.183000
3.146400	-4.082880	12.533696	5.026957	.221566
2.177252	6.341802	-3.326558	7.138753	23.111003
20.488302	7.791042	8.232833	-3.213733	-7.570987
-20.256789	-13.205431	-15.964345	-13.571476	-5.457181
1.234255	-4.812596	-.450077	-2.960061	9.831951
7.665561	10.532449	-8.974041	-7.179233	-13.343386
-3.874709	4.900233	16.320186	13.256149	19.004919
19.003935	8.803148	9.642519	6.514015	1.011212
-12.191030	-6.352824	9.917740	4.134192	-9.092646
-6.874117	-10.299293	-7.439435	-11.351548	-16.481238
-16.384991	-21.907993	-15.326394	-2.461115	-2.568892
-.855114	.115909	-3.307273	-13.245818	-2.196655

12.442676	14.954141	13.963313	4.370650	-6.103480
-10.882784	-5.106227	-2.284982	11.772015	7.817612
3.254089	9.003272	-2.797383	11.562094	-2.350325
1.519740	4.815792	3.052634	3.042107	5.433685
7.346948	4.877559	9.902047	-1.078362	-18.262690
-6.010152	-4.408122	5.473503	17.178802	22.143042
3.714433	-2.028453	.177237	-10.058210	-9.646568
1.482746	-4.213804	-6.171043	-3.536834	-.629467
-8.903574	-12.322859	1.541713	8.233370	7.186696
2.549357	2.039486	-.368412	-2.094729	-13.075783
-4.660627	-5.128501	-19.502801	-11.002241	5.598207
4.878566	-.497147	15.402282	16.321826	12.657461
8.925968	18.540775	-1.567380	-13.453904	-15.763123
-12.410499	-18.528399	-.422719	10.661825	-1.270540
1.183568	4.746854	.797483	-1.162013	5.070389
-3.543689	-.434951	11.252039	6.001631	-3.198695
-7.758956	-11.807165	-12.045732	-10.836585	-24.869268
-6.495415	1.003668	-11.397065	14.482348	14.385878
12.708703	21.566962	21.253570	4.002856	2.202285
-.238172	-13.390538	-6.512430	2.790056	2.032045
13.625636	22.300509	18.440407	19.152325	14.721860
-13.222512	-14.778009	-17.222407	-7.977926	-1.382341
2.894127	3.115302	8.692242	-5.446207	-19.556965
-9.845572	-23.476458	-22.381166	-14.704933	-5.963946
-1.171157	.463074	.170459	-6.063632	-7.050906
-15.840725	2.127420	-2.498064	8.001549	-.198761
2.040991	3.232793	14.186234	1.348988	19.279190
14.223352	1.578682	1.662945	13.130356	.104285
7.683428	5.546742	-1.962606	4.229915	-6.416068
-9.732354	-10.986283	-4.589027	-7.471221	2.223023
9.378418	-5.697265	-10.357812	3.913750	15.731000
7.584800	16.467840	8.974272	9.579418	3.263534
1.210827	-7.831338	-16.065071	-16.052056	-5.041645
-1.033316	-1.426653	13.058678	1.646942	.717554
5.374043	13.099234	2.479387	21.983510	10.786808
.629446	-4.496443	-10.997154	-13.797723	-19.238179
-25.390543	-32.712434	-16.769948	-27.615958	1.107234
10.685787	15.348629	6.878904	18.903123	6.322498
7.857999	4.286399	14.029119	4.623295	-9.701364
-11.761091	-15.008873	-8.607098	-1.685679	5.251457
7.801166	19.840933	20.872746	5.698197	6.158557
11.126346	6.301477	-7.358819	-11.687055	-20.349644
-15.079715	-7.663772	-3.131018	11.295186	5.436149
3.148919	-1.280865	-6.624692	-11.099754	-12.279803
.176158	6.740926	5.592741	17.274193	26.819354
15.655483	17.524387	2.219509	-14.824393	-15.059514
-11.447611	-8.758089	.593529	3.074823	-.140142
6.687887	12.750309	7.000248	-2.399802	13.280158
16.224127	-2.020699	6.183441	.546753	-19.962598
-24.770078	-18.816063	-20.052850	-10.242280	.606176
4.884941	11.307953	8.246362	-2.402910	-12.522328
.582137	-3.334290	2.932568	20.146054	20.116843
6.693475	14.154780	4.123824	-6.700941	-18.760753
-8.208602	-12.166882	-14.733505	-9.186804	2.650557
4.520445	-1.983644	4.813085	5.650468	-4.479626
-.983701	9.013040	-4.789568	-4.031655	7.774676
17.619741	11.295793	5.436634	.749307	1.399446
-18.480443	-9.984355	1.612516	16.090013	4.272010
12.617608	8.694087	14.155269	-7.075785	-5.060628

TIME, 14 MINUTES AND 17 SECONDS

APPENDIX I-D

INVENTORY ON HAND

-120.2000	-114.5600	-97.6480	-72.7184	-76.9747
-87.5798	-59.2638	-49.2111	-48.5688	-24.6551
-11.7241	1.8208	12.8566	26.0853	24.2682
5.8146	24.0517	7.6413	-3.2869	-15.0295
-19.6236	9.5011	2.0009	-6.7993	-4.2394
-19.5916	-19.8732	-30.0986	-4.8789	19.0969
20.2775	-.3780	-11.5024	-24.6019	-13.4815
-2.3852	-2.7082	14.6335	32.3068	28.0454
11.0363	8.0291	-11.3767	-20.9014	-1.3211
3.9431	-10.0455	-16.2364	-44.1891	-45.1513
-5.9210	-7.1368	4.8905	-4.4876	3.0099
-3.9920	-14.5936	-12.8749	9.3001	8.6401
24.7120	5.5696	35.8557	19.4846	15.3877
19.3101	-.7519	-1.4015	-9.5212	-13.2170
12.4264	-1.2589	-1.6071	-11.2857	5.5715
-1.9428	13.2457	28.7966	13.2373	19.1898
26.5519	9.0415	-6.3668	5.3065	-8.7548
-15.6038	-14.2830	-28.4264	-13.7412	-23.9929
-35.5943	-17.0755	-9.6604	16.0717	1.6574
-5.4741	-19.3793	-46.3034	-41.0427	-16.2342
11.0126	6.6101	-7.3119	18.3505	11.2804
17.2243	31.7794	41.6236	24.6988	26.1591
48.9273	34.7418	29.9934	27.3948	35.3158
16.8526	26.8821	10.5057	23.0046	21.4036
8.3229	-14.1417	-20.3133	-19.0507	-10.0405
-23.8324	-16.2659	-39.2128	-36.5702	-48.2562
-15.0049	11.7961	22.2368	12.1895	7.9516
1.9613	-.6310	3.2952	-4.9638	-12.3711
-21.8969	-23.7175	-16.5740	-36.2592	-21.6074
5.5141	6.2113	-9.8310	-12.8648	-11.4918
-5.9935	16.0052	-11.9958	-2.7966	7.3627
-15.3099	-23.6479	-16.1183	-20.0946	-13.8757
7.6994	6.5595	9.6476	15.9181	-5.4655
7.4276	9.7421	-.6063	13.3149	24.0519
23.2416	-5.2068	-14.9654	-8.5723	6.3421
14.0737	2.0590	4.6472	4.9177	12.3342
22.0674	29.0539	19.6431	14.7145	1.5716
-14.5427	1.9658	9.7727	32.2181	24.3745
25.2996	26.6397	-3.2883	-16.6306	-15.9045
-30.3236	-15.4589	-8.9671	-.5737	22.5411
7.4328	-5.0537	-15.4430	-4.7544	1.9965
17.1972	5.5578	-.1538	-12.7230	3.0216
-6.9827	-28.3862	-17.5090	-5.4072	.8743
8.6994	-7.0405	-29.6324	-36.1059	-17.2847
-11.4278	-9.5422	3.3662	15.8930	5.7144
5.9715	16.5772	1.8618	19.4894	17.5915
15.2732	-5.3814	3.8949	7.7159	23.5727
29.8582	20.0865	24.8692	2.2954	-14.9637
-13.5710	-1.2568	10.3946	4.5157	-4.7875
-8.6300	-2.7040	-16.3632	-38.4905	-43.9924
-23.9939	-12.9952	-.5961	-7.0769	-7.8615
-16.4892	-3.9914	-11.7931	-.6345	16.4924

3.3939	-.2849	3.9721	13.9777	23.9822
8.3857	4.9086	32.7269	34.9815	41.7852
23.0282	8.6225	-10.7020	-15.9616	-40.7693
-54.8154	-36.8523	-38.6819	-35.5455	-27.8364
-21.8691	-18.0953	9.9238	8.7390	.1912
1.7530	10.0024	7.2019	17.5615	-9.9508
11.6394	13.5115	11.4092	6.7274	15.1819
22.9455	15.3564	3.4851	10.7881	18.4305
17.1444	18.9155	1.5324	-15.3741	-6.4993
-17.3994	-8.3195	-16.2556	-6.6045	-17.4836
-27.5869	-31.0695	-11.0556	-1.2445	-8.3956
-5.1165	18.9068	9.7255	-4.6196	-6.8957
-.7166	1.4267	3.9414	7.9531	20.9625
32.1700	-9.0640	-16.4512	-23.1610	-5.5288
5.5770	8.4616	1.3693	-.3046	-14.8437
-15.2749	-25.2199	-.7760	22.3792	19.5034
21.4027	23.7222	52.5777	32.6622	36.9297
32.3438	31.0750	20.2600	10.6080	-1.7136
-10.3709	-33.6967	-44.5574	-34.0459	-32.2367
-34.1894	-35.5515	-34.4412	-.5530	16.9576
15.9661	15.9729	1.9783	.3826	-5.0939
7.5249	5.8199	1.0559	-1.9553	-12.3642
-28.8914	-10.7131	10.8295	-3.9364	-.1491
.0807	17.0646	21.4517	-4.6387	-8.9109
5.0712	27.6570	9.3256	-.1395	1.0884
-16.9293	-15.1434	-12.7147	7.6282	21.9026
3.9220	-19.4624	-26.5699	-20.2559	-22.2047
-1.9638	1.0290	16.8232	22.2585	30.8068
22.4455	25.3564	37.8851	28.9081	17.7265
2.7812	-11.7751	-14.6201	-2.6960	-22.5568
-8.8455	-11.8764	-27.3011	-22.0409	5.3673
2.2938	-1.1649	11.6681	3.7344	-10.0124
-8.4100	-7.9280	-29.7424	-29.7939	-16.4351
-17.5481	-.4385	3.8492	5.6794	31.3435
22.0748	15.4598	18.9679	15.1743	-14.2606
-29.6084	-13.4868	-16.9894	-14.5915	1.3268
-10.5386	-18.8309	3.9353	33.3482	37.6786
64.5429	46.2343	44.9874	39.9900	26.9920
-.4064	5.0749	-1.9401	-5.1521	-13.9217
-15.5373	-27.4299	-7.3439	-8.8751	-16.7001
-15.9601	-21.9681	-6.3744	16.9004	11.5204
26.8163	4.8530	22.6824	16.9459	14.5567
13.2454	22.3963	-11.8829	-33.5064	-31.0051
-13.8041	-8.8433	5.5254	10.2203	-11.6237
-3.0990	-4.0792	-16.4634	-23.3707	-28.2965
-29.4372	-52.5498	-44.2398	-40.5919	-25.4735
-12.3788	-12.5030	-.4024	13.2781	18.0224
35.6180	44.6944	66.7555	58.4044	53.5235
20.0188	-4.3850	-5.1080	8.1136	-14.3091
-26.6473	-27.1178	-11.0943	-26.8754	-14.7003
-13.1603	-5.5282	-6.6226	-11.8981	-19.9184
-11.7348	12.0122	31.2098	20.3678	32.8942
9.5154	13.4123	-4.4701	.6239	14.2991
16.0393	-9.7686	13.3851	5.3081	11.8465

-15.1228	5.9018	-24.0786	-7.2629	-5.8103
-28.2482	-10.5986	-8.8789	-15.1031	12.3175
8.8540	21.0832	15.6666	10.7333	7.9866
-5.6107	-10.2886	-11.0309	6.9753	-2.0197
-18.4158	-19.3326	-33.8661	-19.4929	-18.9943
-30.9954	-34.5964	-4.4771	-7.5817	-3.2653
14.5877	12.8702	-11.1039	-1.2831	-14.4265
15.2588	25.8071	34.0456	28.8365	-2.7308
-3.7846	2.7723	5.8178	5.8543	-1.9166
4.0667	4.0534	.0427	-11.7658	-29.8127
-21.6501	-23 3201	-33.0561	-20.0449	-5.8359
-19.0687	-10 0550	-16.6440	-25.1152	-17.8921
-18.7137	8290	-.5368	15.7706	12.8165
19.4532	12.9625	23.5700	19.4560	37.1648
28.9319	25.1455	.9164	-1.8669	-6.8935
5.8852	18.1082	29.0865	7.8692	12.6954
9.3563	-7.7150	-18.1720	1.6624	-2.0701
-8.0560	15.1552	29.1241	25.8993	24.1194
23.8956	29.3164	32.4532	18.5625	19.2500
-13.2000	-23.9600	-34.1680	-42.7344	-36.1875
-42.3500	-27.6800	-17.7440	-28.9952	7.8038
-5.3569	14.1145	8.6916	10.7533	19.4026
13.5221	2.2177	26.5741	16.8593	26.4874
22.1900	22.3520	4.8816	6.3053	27.8442
15.6754	-1.8597	-3.0878	-26.0702	-23.4562
-39.5649	-23.6519	-27.3216	-21.2572	-2.6058
7.5154	-5 9877	-5.3902	-9.7121	8.8303
14.4642	17.5714	1.2571	7.8057	4.8446
15.4756	13.3805	40.7044	37.7635	34.0108
25.2087	6.7669	21.4135	13.5308	-9.7753
-27.2203	-23.9762	1.6190	-9.5048	-38.4038
-34.3231	-40.4584	-26.9668	-27.5734	-39.0587
-35.2470	-38.3976	-29.5181	-7.8145	-1.8516
6.3187	14.6550	-1.6760	-11.5408	-4.0326
11.3739	25.8991	15.5193	23.2154	-4.4277
-13.5421	-.0337	-1.6270	33.4984	12.1987
10.7590	15.8072	2.0458	30.0366	1.4293
12.7434	9.3947	10.1158	1.6926	.9541
12.1633	3 7306	23.7845	12.0276	-13.7779
-.6223	-6.6979	8.2417	19.3934	30.7147
7.1718	-13.2626	-11.4101	-21.5281	-18.4224
-6.3380	-15.4704	-10.9763	2.0190	1.0152
-12.3879	-13.5103	.3918	5.5134	.0107
-1.3914	-8.5131	-19.0105	-12.2084	-17.5667
-7.8534	-3.8827	-14.5062	3.3951	23.9161
22.1328	10 1063	20.8850	21.3080	8.0464
.2371	11.1897	-7.2482	-16.5986	-24.8789
-20.7031	-30.1625	.0700	24.2560	2.0048
-.1962	10.4431	5.5545	9.4436	-1.2451
-18.1961	-7.5569	-3.2455	-3.7964	-28.8371
-24.4697	-23.3758	-25.3006	-4.6405	-25.1124
9.5101	23.4081	.3265	34.0612	30.4489
21.5591	28.4473	28.7579	9.8063	16.8450
15.6760	.3408	7.6727	24.3381	15.4705

16.5764	22.4611	19.1689	26.7351	14.3881
-19.8895	-25.1116	-26.6893	-23.5514	-22.0411
-12.4329	-20.3463	-10.4771	-27.7817	-39.8253
-29.0603	-43.0482	-43.4386	-30.1509	-17.7207
-12.9765	-2.7812	-.4250	-9.1400	-11.3120
-15.6496	11.8803	4.5043	24.2034	16.3627
21.8902	15.5121	26.2097	17.7678	35.6142
35.0914	9.0731	1.8585	18.2868	-8.3706
8.5035	3.6028	-14.1177	-6.8942	-8.5153
-12.2123	-18.3698	-14.2959	-6.6367	18.2907
27.8325	9.2660	-3.9872	15.0103	29.6082
14.2866	16.4292	1.3434	2.4747	-1.4202
-3.5362	-11.4289	-24.1432	-21.1145	-1.0916
9.3267	11.6614	29.5291	21.2233	12.7786
14.0229	11.8183	-3.5453	12.1637	-10.2690
-30.6152	-42.4922	-45.1937	-45.1550	-37.3240
-35.4592	-47.3674	-16.6939	-19.9551	13.6359
28.7087	31.1670	25.7336	35.3869	1.1095
-2.7124	-5.1699	20.4641	7.5712	-15.3430
-12.2744	-.4195	6.8644	12.6915	16.9532
22.5626	39.6501	22.9200	-.0640	-9.6512
2.0791	3.2632	-16.3894	-13.7115	-33.9692
-26.5754	-10.8603	-10.6882	5.4494	1.1595
2.7276	7.7821	4.2257	.9805	3.9844
17.7875	26.6300	16.5040	20.0032	28.8026
14.4421	19.3537	5.4829	-26.4137	-26.7309
-6.5847	-4.8678	9.7058	14.3646	14.8917
23.3134	22.2507	13.4005	-6.4796	-.3837
-1.3069	-24.2455	-10.9964	-8.3971	-41.1177
-39.4942	-23.3953	-31.9163	-16.3330	-4.0664
7.3469	21.6775	21.1420	18.3136	-6.1491
12.0807	11.0646	9.6516	25.3213	14.6571
1.9256	9.3405	-9.9276	-20.3421	-34.0737
-16.4589	-16.7671	-19.0137	-13.2110	.4312
8.5450	5.2360	6.7888	9.4310	7.5448
13.0359	21.8287	-3.3371	1.1304	10.7043
15.5634	10.4507	19.5606	13.8485	2.6788
-20.4570	.4344	18.7475	21.9980	

TIME, 14 MINUTES AND 35 SECONDS

APPENDIX I-E

INVENTORY ON HAND

-342.0000	-299.8000	-238.4400	-167.5520	-169.0416
-179.2333	-137.1866	-121.3493	-132.0794	-94.8636
-45.6908	-34.3527	-13.0821	17.1343	17.5074
2.8059	12.6448	11.5158	9.4126	-8.0699
-14.0559	26.5553	14.8442	-13.1246	-24.4997
-53.5998	-35.4798	-32.7838	18.3729	41.6983
42.9587	4.9669	-24.6265	-51.1012	-38.6809
-19.5447	-24.6358	8.6914	44.3531	42.2825
20.6260	30.5008	-4.3994	3.4805	31.3844
10.3075	-16.9540	-27.1632	-60.5305	-67.2244
-11.7796	9.1764	28.9411	-1.8471	10.1223
-14.5022	-20.4017	-46.3214	-6.8571	5.1143
17.4915	-3.6068	40.7145	30.9716	21.3773
15.1018	8.0815	.2652	-1.9879	-9.9903
31.8078	-1.7538	-12.4030	-22.7224	-5.9779
-33.3823	-8.9059	29.8753	.3002	15.4402
41.1522	27.3217	29.8574	29.4859	11.7887
5.2310	-15.8152	-49.0522	-35.4417	-33.5534
-51.4427	-22 7542	24.1967	56.7573	17.0059
-4.5953	-15.6762	-55.3410	-68.0728	-13.0582
15.7534	-1.7973	-36.0378	12.3697	-6.1042
-9.2834	17.7733	39.6186	17.0949	13.4759
51.3807	24.1046	32.0837	30.0669	32.2536
11.2028	30.9623	12.3698	21.6959	25.9567
31.3653	-2.5077	1.5938	8.8751	34.3000
-6.7600	-17.8080	-60.4464	-58.7571	-86.6057
-41.6845	9 0524	14.8419	8.8735	18.0988
14.8790	11.3032	13.8426	16.6741	3.3393
-16.3286	-33.8629	-14.2903	-36.0322	-25.8258
4.5394	-6.7685	-13.6148	-18.0918	-20.6735
.8612	44.6890	.1512	7.5209	20.0168
-23.3866	-39.3093	-22.4474	-27.9579	-1.7663
10.9869	-4.2105	7.8316	7.4653	-32.0278
-20.4222	10.6622	4.9298	32.9438	47.3551
42.0841	3.0672	-23.7462	-22.7970	-14.0376
-6.8301	-24.2640	3.1888	3.7510	28.8008
60.0406	48.8325	22.2660	-5.9872	-28.3898
-52.7118	-29.7694	7.3844	59.7076	40.7660
56.2128	51 1703	8.3362	-15.1310	-27.3048
-46.2439	-23.1951	-9.3561	1.5151	26.2121
-11.4303	-11 7442	-18.7954	8.3637	14.2909
34.4328	21.3462	-2.7230	-23.5784	-3.2627
-26.8102	-41.0482	1.3615	24.6892	22.3513
40.0811	7.0649	-35.5481	-64.0385	-33.6308
-31.5046	-35.8037	.3570	21.2856	12.4285
7.9428	33.9542	-4.2366	13.2107	-1.0314
-4.8251	-20.8601	-8.2881	9.9695	44.3756
57.5005	27.6004	17.0803	-18.1357	-27.3086
-22.8469	-2.2775	22.5780	27.4624	18.3699
10.2959	9.8367	-12.5306	-37.8245	-52.6596
-19.1277	-16.7021	7.6383	-9.2894	-29.6315
-40.1052	-2.8842	-12.1073	8.5141	24.8113

14.2490	5.9992	-9.8006	-10.8405	-11.2724
-18.4179	-22.5343	30.1725	44.9380	80.1504
66.3203	48.6563	22.1250	18.5000	-29.4000
-53.7200	-30.7760	-51.0208	-63.8166	-40.2533
-30.4026	-34.9221	6.2623	.2098	12.9679
-6.2257	3.8194	5.0556	27.8444	-22.1244
7.7004	10.5604	8.0483	-3.1614	5.4709
22.9767	2.9814	-5.2149	18.4281	22.1425
23.3140	28.8512	18.4809	-17.8152	-4.2522
.1982	14.9586	-15.6331	-8.7065	-16.7652
-29.8122	-51.2497	-11.5998	16.5202	7.6161
-11.3071	20.9543	-.2365	-18.1892	-25.3514
-16.8811	9.8951	13.5161	20.0129	27.6103
31.4882	-30.6094	-24.0875	-34.6700	.6640
20.3312	48.2649	14.6120	-1.9104	-24.1283
-22.9027	-39.5221	-26.4177	15.8658	6.2927
-1.3659	-5.6927	39.6458	12.7167	28.3733
27.4987	49.5989	59.0792	51.0633	32.8507
26.2805	-14.3756	-30.7005	-41.7604	-43.6083
-52.0866	-65.6693	-56.1354	-9.9084	15.2733
8.2187	17.7749	-2.5801	-1.8641	-1.8912
34.6870	27.5496	-2.5603	1.1517	-9.4786
-27.9829	-19.7863	-2.0290	-10.2232	-4.9786
-9.5829	-.2663	3[illegible].3870	7.3096	-9.1523
14.8781	44.7025	1[illegible].1620	-18.2704	-24.0163
-38.4131	-12.3304	-1.6644	35.4685	62.7748
26.4198	-5.8641	-29.8913	-26.9130	-51.5304
-21.0243	-17.2195	-3.7756	-6.8205	20.5436
14.0349	19.6279	44.7023	40.7619	41.2095
10.5676	3.2541	14.4033	23.7226	-27.2219
-6.1775	-20.7420	-58.9936	-54.7949	-.8359
-1.6687	-7.5350	29.7720	40.0176	14.2141
16.9713	14.3770	-24.0984	-41.0787	-52.0630
-42.2504	-18.0003	-20.4002	-14.5202	35.7838
31.6271	16.9017	39.7213	40.9771	5.7817
-15.9747	19.2203	-4.6238	-30.4990	-22.9992
-59.5994	-67.6795	-37.5436	8.1651	16.7321
80.7257	61.4285	63.7428	57.7943	45.4354
2.3483	16.4787	2.5829	-.1337	-1.5069
-12.2055	-8.1644	13.4685	-4.8252	-17.8602
-27.6882	-41.1505	-44.3204	-5.0563	-8.2451
16.8039	-15.7568	39.5945	41.0756	45.6605
42.1284	50.5027	-16.1978	-58.1583	-42.9266
-30.5413	-22.4330	8.2536	25.2029	-4.0377
2.7698	19.4159	2.5327	-8.1738	-11.1391
-9.5113	-33.8090	-33.8472	-35.4778	-29.7822
-37.2258	-51.7806	-47.2245	-39.9796	-37.3837
16.0931	56.6744	90.7396	76.5916	91.0733
48.8587	6.2869	-8.1705	20.6636	-21.6691
-31.3353	-21.4682	-.1746	-15.3397	2.3283
4.4626	-14.6299	-31.1039	-34.6831	-50.5465
-40.6372	8.0902	50.0722	35.2577	40.4062
10.9250	21.3400	-16.3280	-12.6624	-4.5299
23.3761	-27.6992	27.0407	17.2325	25.5860

6.4688	28.1751	-20.4600	7.4320	-13.2544
-45.6035	-30.6828	-20.5462	-30.6370	7.2904
12.6323	25.7059	26.3647	7.4918	-.8066
-12.0453	.9638	7.1710	32.7368	21.9895
4.7916	-5.9667	-47.5734	-25.4587	-13.3670
-41.0936	-48.6749	9.8601	-13.3119	-1.6495
3.4804	-4.2157	-44.9726	-28.1780	-23.7424
25.2060	40.3648	47.0919	27.2735	-6.5812
-16.8650	-9.8920	.2864	18.6291	15.9033
24.1226	26.8981	26.9185	2.5348	-41.3722
-17.4977	-32.5982	-38.2785	-14.4228	11.8617
-5.9106	-1.5285	-7.8228	-23.2582	-20.2066
-35.7653	-11.8122	-21.2498	-.9998	-14.5999
10.9201	9.9361	28.9489	30.7591	68.4073
36.5258	14.2207	-22.6235	-18.4988	-38.3990
-8.9192	27.4646	55.5717	10.2574	30.6059
36.0847	-3.9322	-30.1458	2.4834	-15.2133
-32.7706	-8.0165	3.3868	-.4906	-4.5925
22.1260	44.3008	55.4407	59.5525	54.4420
24.1536	-3.8771	-19.5017	-24.4013	-46.5211
-54.6169	-37.4935	-26.7948	-49.4358	-1.7487
-15.7989	13.5609	-2.1513	17.6789	25.3432
8.2745	-8.5804	31.5357	14.0286	-.7772
3.9783	13.1826	-11.2539	12.1969	53.5575
48.6460	19.9168	19.7334	-8.0132	-16.2106
-48.3685	-30.4948	-37.5958	-29.2767	-12.8213
1.3429	-14.1257	-4.5005	-11.6004	23.1197
18.0957	23.0766	-22.7387	-19.5910	-35.2728
-10.4182	13.6654	40.7323	35.1859	51.5487
50.4390	23.5512	27.4409	18.7527	4.8022
-28.7582	-13.6066	27.1147	11.6918	-24.2466
-19.3973	-28.1178	-20.0942	-29.0754	-40.6603
-40.9283	-56.7426	-42.1941	-10.3553	-8.8842
-3.1074	1.7141	-4.6287	-30.9030	-2.5224
32.7821	38.4257	33.9405	8.5524	-17.5581
-29.4464	-15.9572	-6.7657	31.7874	21.0299
9.8239	24.4592	-5.2327	29.0139	-5.7889
2.7689	11.0151	5.2121	5.7697	12.2157
16.7726	7.4181	23.5345	-2.1724	-41.9379
-12.3504	-8.2803	13.3758	41.3006	51.6405
7.1124	-4.3101	4.3519	-25.3185	-23.0548
2.3562	-13.1150	-19.6920	-10.1536	-2.7229
-22.7783	-30.8227	2.7419	17.3935	15.3148
5.2518	6.4015	1.7212	-.4231	-28.3384
-9.0708	-13.8566	-48.0853	-28.2682	10.3854
10.9083	.1267	39.9013	44.3211	36.6569
25.1255	46.7004	-4.2397	-35.9918	-39.1934
-29.9547	-44.9638	-1.3710	28.3032	-2.7575
2.7940	9.6352	1.3082	-4.1535	9.8772
-11.6982	-2.5586	24.3531	12.0825	-9.1340
-19.5072	-30.2058	-30.3646	-28.4917	-61.5933
-16.2747	4.3803	-24.2958	39.1634	37.7307
34.5846	54.6676	51.5341	10.8273	[illegible]
-.8305	-34.4644	-18.5715	4.9428	3.7542

31.4034	56.3227	48.6582	50.3265	38.6612
-29.6710	-36.1368	-43.1095	-19.6876	-2.9501
7.0400	9 4320	21.5456	-11.9635	-49.9708
-23.7767	-59.0213	-55.4171	-38.9337	-13.5469
-4.8375	1.7300	-.2160	-13.9728	-18.1783
-37.7426	5.4059	-4.2753	19.1798	.1438
5.1151	8.0921	33.8736	3.6989	47.3591
33.6873	3.1498	6.7199	32.9759	-1.2193
16.6246	12.6997	-7.8403	7.5278	-17.1778
-24.3422	-28.4738	-12.9790	-21.1832	4.6534
23.5227	-13.1818	-24.5454	10.3636	39.6909
16.7527	41.6022	24.2817	25.6254	8.1003
5.4803	-17.2158	-38.7726	-36.6181	-8.6945
-2.1556	-3.3245	34.3404	6.6723	3.1379
17.3103	34.4482	6.9586	55.3669	24.4935
-1.6052	-15.0842	-32.0673	-38.8539	-49.8831
-64.9065	-80.3252	-38.2601	-64.8081	4.9535
26.5628	36.0502	14.4402	46.1522	15.1217
20.2974	12.0379	34.6303	11.5043	-22.9966
-26.3973	-36 3178	-19.8543	-2.6834	14.4533
18.7626	47.0101	49.8081	14.4465	12.9572
23.9657	12.9726	-20.8219	-34.4575	-56.7660
-40.8128	-21 2503	-9.0002	28.7998	16.2399
9.9919	-.4065	-11.5252	-22.6202	-26.8961
1.0831	15.0665	10.4532	37.9625	61.1700
35.5360	42.2288	6.3831	-33.4936	-31.9948
-22.9959	-19.1967	3.0426	8.2341	.3873
16.7098	34.1679	18.9343	-6.4526	31.0379
38.8304	-8.9357	15.0514	2.6411	-47.4871
-58.3897	-42.9117	-46.5294	-25.8235	-1.2588
8.9930	28.1944	18.9555	-5.0356	-27.0285
3.7772	-7.7782	6.3774	49.5019	47.4015
14.9212	31.5370	8.0296	-16.5763	-45.4611
-20.9688	-30.1751	-39.5401	-28.4321	3.0544
11.0435	-3.1652	15.2678	20.6143	-7.1086
.1131	22.2905	-11.3676	-12.2941	15.1647
39.3318	26.0654	13.6523	2.5219	5.2175
-45.0260	-23.8208	4.1434	39.5147	10.2118
31.7694	23.0155	36.2124	-15.6301	-10.9041

TIME, 14 MINUTES AND 17 SECONDS

APPENDIX I-F

INVENTORY ON HAND

-56.000	-63.000	-57.600	-41.480	-57.184
-75.147	-63.518	-64.014	-76.211	-63.169
-40.535	-39.628	-29.503	-15.002	-16.602
-26.281	-23.225	-25.580	-27.064	-38.051
-41.041	-16.033	-22.626	-42.101	-50.081
-68.465	-56.372	-54.497	-19.198	-4.158
-5.127	-31.101	-53.081	-72.465	-64.172
-49.538	-52.830	-27.864	-2.091	-2.273
-19.018	-10.815	-37.452	-30.161	-10.129
-27.303	-49.643	-58.314	-85.051	-88.841
-43.473	-26.578	-11.663	-37.930	-28.744
-49.795	-53.436	-75.549	-41.439	-31.351
-19.681	-37.345	2.924	-4.461	-13.969
-21.175	-26.940	-34.752	-38.002	-46.001
-7.201	-38.561	-49.249	-57.599	-40.079
-65.663	-41.531	-3.025	-32.820	-19.856
5.915	-7.868	-5.294	-6.635	-23.308
-29.847	-51.877	-85.102	-70.481	-70.785
-88.628	-57.903	-9.122	25.302	-15.758
-38.606	-49.885	-93.708	-108.167	-49.933
-20.147	-38.917	-74.334	-21.067	-42.054
-45.443	-14.754	8.597	-16.523	-20.418
23.265	-8.588	2.330	.264	1.411
-22.671	.463	-21.830	-10.664	-3.731
2.015	-37.388	-31.310	-22.848	6.721
-40.423	-54.338	-107.071	-106.656	-138.725
-84.780	-24.024	-16.219	-23.375	-12.100
-17.280	-20.624	-18.699	-13.560	-31.448
-54.358	-76.286	-51.829	-81.063	-68.651
-30.521	-45.016	-51.213	-57.571	-61.056
-32.645	23.884	-34.893	-24.514	-8.411
-65.729	-86.383	-64.707	-71.165	-37.132
-21.906	-43.125	-25.700	-25.960	-79.768
-64.414	-20.131	-28.105	9.316	29.253
22.802	-31.358	-67.287	-65.829	-52.663
-43.731	-68.985	-31.588	-30.670	3.864
49.291	35.633	-3.294	-42.835	-73.268
-109.414	-78.131	-24.705	50.036	21.629
43.703	37.562	-23.350	-57.080	-74.064
-102.251	-70.001	-48.801	-32.041	4.367
-50.106	-50.685	-62.348	-20.078	-12.263
16.990	-2.408	-37.326	-69.461	-38.769
-75.415	-95.932	-33.546	2.363	-.309
25.553	-25.158	-89.726	-132.581	-87.065
-82.452	-87.562	-32.649	-2.119	-15.695
-25.156	15.275	-43.180	-16.144	-38.715
-42.572	-68.058	-46.446	-17.957	36.834
59.068	12.654	-7.077	-62.061	-77.649
-71.519	-38.615	2.908	9.126	-5.899
-20.719	-20.975	-57.780	-96.824	-120.059
-67.248	-63.398	-22.918	-51.135	-83.308
-97.246	-36.597	-54.478	-21.782	5.174

-15.061	-28.248	-52.999	-53.399	-52.519
-62.015	-71.212	18.230	40.984	100.987
75.190	44.952	-.438	-4.951	-86.361
-127.689	-86.951	-120.761	-143.609	-103.687
-85.349	-94.480	-22.984	-34.787	-11.830
-46.064	-27.851	-26.081	13.735	-74.612
-22.489	-18.391	-24.113	-42.491	-26.592
5.326	-30.539	-43.231	-.785	2.972
5.978	16.582	-3.534	-66.827	-41.062
-34.450	-8.160	-62.328	-51.862	-65.690
-87.752	-126.001	-54.201	-4.761	-21.809
-58.047	2.962	-36.430	-69.744	-81.795
-64.636	-16.709	-8.767	4.786	18.429
24.943	-91.245	-77.796	-98.637	-33.710
2.232	52.986	-11.011	-40.009	-78.607
-74.886	-108.909	-83.527	-5.422	-24.537
-39.830	-45.464	40.829	-9.937	20.051
17.040	60.032	77.626	62.301	27.441
15.752	-61.198	-92.958	-113.167	-117.933
-134.747	-159.597	-142.278	-52.422	-3.338
-18.670	-2.536	-42.029	-40.623	-39.699
30.241	17.993	-41.406	-35.325	-55.860
-92.288	-77.430	-40.344	-56.275	-45.420
-54.336	-34.469	27.025	-19.780	-52.424
-3.139	55.689	-.049	-69.639	-80.711
-111.169	-60.535	-39.628	34.297	89.638
16.510	-46.592	-97.273	-92.819	-140.455
-76.964	-70.171	-40.937	-45.950	6.840
-4.928	6.458	57.966	49.373	52.698
-9.241	-25.393	-2.314	16.148	-88.881
-47.705	-79.764	-160.011	-151.209	-40.967
-41.574	-52.459	25.833	45.666	-7.467
-1.374	-8.099	-86.679	-123.143	-147.515
-126.412	-73.529	-79.423	-65.739	42.609
33.487	2.590	49.872	51.897	-22.882
-68.306	6.555	-42.956	-96.564	-82.452
-161.561	-180.249	-115.199	-18.559	1.353
137.082	96.666	102.932	90.746	64.997
-27.003	1.198	-30.642	-38.913	-41.331
-64.465	-52.972	-6.377	-45.102	-70.481
-92.385	-123.508	-130.407	-42.725	-52.780
1.976	-68.819	53.745	56.996	67.797
56.237	74.790	-72.968	-163.975	-130.180
-102.344	-85.675	-17.940	21.848	-41.122
-24.497	11.402	-26.678	-52.943	-60.354
-56.683	-111.147	-114.517	-118.214	-104.571
-120.657	-154.325	-142.660	-124.328	-117.663
1.670	93.536	170.029	138.623	172.898
79.119	-17.105	-50.884	14.093	-84.726
-104.981	-85.184	-35.748	-71.998	-30.998
-26.599	-70.679	-108.743	-113.995	-150.796
-127.837	-13.269	83.185	47.348	60.278
-9.377	14.698	-74.042	-67.233	-48.587
19.131	-99.695	28.444	5.355	26.484

-19.613	28.310	-88.352	-18.282	-67.625
-141.100	-104.080	-77.664	-104.331	-14.265
-3.612	23.910	26.328	-19.137	-40.310
-65.848	-33.278	-19.823	41.742	15.193
-24.045	-49.636	-151.509	-96.007	-65.606
-133.685	-150.948	-9.958	-64.167	-36.933
-22.947	-41.957	-141.966	-104.173	-91.338
28.930	63.544	79.835	31.868	-50.306
-78.445	-59.756	-35.205	9.436	4.549
26.239	32.191	28.753	-31.997	-139.998
-80.198	-114.359	-125.687	-68.950	-3.360
-48.288	-40.830	-57.264	-93.611	-87.889
-124.511	-64.009	-86.607	-39.686	-74.149
-13.319	-14.055	34.556	41.445	137.156
58.525	1.220	-90.224	-78.379	-131.104
-58.083	32.134	104.707	-11.434	40.852
58.882	-42.094	-110.076	-27.260	-71.808
-117.847	-52.277	-21.422	-33.937	-46.350
18.520	72.216	98.373	112.498	100.199
24.959	-44.633	-83.906	-97.525	-155.020
-176.816	-132.453	-101.962	-160.170	-40.936
-76.149	-.319	-41.855	10.116	30.093
-12.926	-55.341	49.327	1.262	-35.190
-21.952	.438	-65.049	-1.340	106.328
93.463	19.370	19.896	-51.483	-75.186
-161.149	-116.719	-137.375	-116.900	-72.520
-34.816	-72.653	-43.922	-62.738	25.810
15.448	29.758	-91.993	-85.795	-126.836
-59.869	1.305	71.044	53.235	96.588
92.271	20.816	32.653	14.323	-23.742
-109.794	-69.835	40.332	-4.334	-96.667
-85.134	-107.507	-85.006	-109.805	-142.244
-145.595	-187.876	-152.301	-65.041	-61.432
-44.546	-31.037	-46.829	-116.864	-43.891
49.487	65.790	53.832	-12.134	-81.508
-112.606	-74.085	-50.868	51.506	22.805
-5.356	33.715	-44.628	45.898	-47.682
-28.546	-6.636	-24.709	-20.367	-.894
13.485	-9.812	29.750	-41<000	-149.400
-67.520	-56.816	4.547	81.238	106.590
-17.128	-49.502	-27.202	-107.561	-100.649
-30.119	-70.895	-85.716	-62.173	-43.138
-96.311	-116.049	-26.239	14.809	9.047
-19.962	-19.170	-29.736	-36.189	-112.751
-61.201	-72.161	-168.728	-113.383	-3.106
-.685	-33.348	74.722	86.377	62.502
28.801	92.241	-46.807	-132.246	-140.597
-113.877	-158.702	-38.561	42.951	-46.039
-28.831	-7.865	-30.292	-46.234	-3.787
-66.430	-42.944	34.245	1.196	-58.843
-89.875	-118.500	-120.600	-115.680	-211.144
-81.715	-24.572	-105.058	74.154	68.523
57.618	118.295	112.036	-4.171	-22.937
-37.750	-134.800	-91.440	-23.752	-23.001

58.799	127.239	102.391	109.513	73.410
-125.672	-140.737	-158.590	-91.472	-41.178
-10.142	-7.714	27.229	-69.617	-180.293
-107.435	-208.348	-196.078	-147.263	-74.610
-47.488	-27.990	-33.792	-75.034	-90.627
-145.102	-18.681	-46.345	25.324	-27.541
-16.633	-11.306	65.555	-26.356	101.315
61.252	-26.598	-18.679	61.457	-40.834
13.933	5.346	-54.323	-10.459	-83.567
-105.053	-118.243	-71.394	-94.715	-17.972
36.022	-74.382	-107.706	-5.365	82.308
12.247	86.597	36.478	43.182	-10.054
-17.443	-88.155	-154.324	-146.259	-64.007
-44.806	-45.845	66.324	-17.541	-28.832
16.934	67.347	-14.122	127.902	38.322
-41.143	-80.514	-128.611	-146.289	-182.831
-230.065	-274.252	-151.202	-231.761	-19.209
46.633	76.106	12.285	105.228	11.182
25.946	-1.443	68.845	.676	-103.659
-115.527	-142.822	-93.257	-42.606	7.115
19.492	104.594	115.275	8.420	6.336
42.669	10.735	-94.612	-137.090	-206.472
-161.777	-105.222	-65.577	47.538	7.430
-8.656	-38.725	-75.180	-106.144	-118.915
-30.132	14.294	-1.164	84.068	157.455
76.964	93.771	-16.783	-142.227	-135.981
-107.385	-92.708	-23.966	-8.573	-35.258
16.793	69.035	23.428	-51.658	64.874
85.099	-64.121	7.303	-33.557	-189.446
-221.357	-170.485	-179.388	-112.511	-33.409
-1.527	53.979	25.383	-52.294	-123.635
-25.108	-60.086	-17.069	120.945	116.756
14.405	67.924	-3.261	-85.409	-176.727
-103.782	-130.025	-160.820	-122.856	-21.285
2.172	-45.062	11.950	27.960	-58.232
-34.386	37.892	-71.687	-75.149	11.480
91.984	47.788	8.630	-23.696	-15.757
-176.205	-111.564	-20.851	92.519	-5.385
62.092	36.074	74.859	-90.113	-70.490

TIME, 14 MINUTES AND 37 SECONDS

APPENDIX II-A

```
      PROGRAM MEAN
      DIMENSION R(1000),E(1000)
C THIS PROGRAM SIMULATES THE BASIC MODEL WITH MEAN DEMAND INPUT
      DO 1 I=1,1000
    1 R(I)=16.0
      DO 4 I=6,1000
      SUM=0.0
      M=I-5
      DO 2 J=1,M
      TEMP1=R(I)*(.8)**J
    2 SUM=SUM+TEMP1
      TEMP1=SUM
      SUM=0.0
      DO 3 J=1,4
      TEMP2=R(I)
    3 SUM=SUM+TEMP2
      TEMP2=SUM
    4 E(I)=TEMP1-TEMP2
      PRINT 5
    5 FORMAT(11HMEAN VALUES)
      PRINT 6,(E(I),I=6,1000)
    6 FORMAT(6X,5F10.6)
```

APPENDIX II-B

```
      PROGRAM MEAN
      DIMENSION R(1000),E(1000)
C THIS PROGRAM SIMULATES MODEL WITH MEAN DEMAND INPUT
      DO 1 I=1,1000
    1 R(I)=1
      DO 4 I=6,1000
      SUM=0.0
      M=I-5
      DO 2 J=1,M
      TEMP1=R(I-4-J)*(.8)**(J-1)
    2 SUM=SUM+TEMP1
      TEMP1=SUM
      SUM=0.0
      DO 3 J=1,5
      TEMP2=R(I+1-J)
    3 SUM=SUM+TEMP2
      TEMP2=SUM
    4 E(I)=TEMP1-TEMP2
      PRINT 5
    5 FORMAT(11HMEAN VALUES)
      PRINT 6,(E(I),I=6,1000)
    6 FORMAT(6X,5F13.6)
      END
      END
```

APPENDIX II-C

```
      PROGRAM POISSON
C THIS PROGRAM SIMULATES BASIC MODEL WITH RANDOM DEMAND
      DIMENSION T(1000),X(1000),IX(1000)
      COMMON IRANDOM,RANDOM
      IRANDOM=3574655
      DO 1 I=1,1000
      CALL NORMAL(16.0,4.0,X(I))
      IX(I)=X(I)
    1 X(I)=IX(I)
      DO 4 I=6,1000
      SUM=0.0
      M=I-5
      DO 2 J=1,M
      TEMP1=(.8)**(J-1)*X(I-4-J)
    2 SUM=SUM+TEMP1
      TEMP1=SUM
      SUM=0.0
      DO 3 J=1,5
      TEMP2=X(I-J+1)
    3 SUM=SUM+TEMP2
      TEMP2=SUM
    4 T(I)=TEMP1-TEMP2
      PRINT 5
    5 FORMAT(24HRANDOM NUMBERS GENERATED)
      PRINT 6,(X(I),I=1,1000)
    6 FORMAT(6X,5F10.6)
      PRINT 7
    7 FORMAT(17HINVENTORY ON HAND)
      PRINT 8,(T(I),I=6,1000)
    8 FORMAT(6X,5F10.6)
      END
      SUBROUTINE RANVAR
      COMMON IRANDOM,RANDOM
      CON(K1=67108864)
      IRANDOM=IRANDOM*3125
      IF(IRANDOM-67108863)2,2,1
    1 ENQ(0),LDA(IRANDOM),DVI(K1),STQ(IRANDOM)
    2 Y=IRANDOM
      RANDOM=Y/67108864.0
      RETURN
      END
      SUBROUTINE NORMAL(A,B,C)
      COMMON IRANDOM,RANDOM
      SUM=0.0
      DO 1 I=1,12
      CALL RANVAR
    1 SUM=SUM+RANDOM
      X=SUM-6.0
      C=X*B+A
      RETURN
```

APPENDIX II-D

```
      PROGRAM POISSON
C THIS PROGRAM SIMULATES BASIC MODEL WITH RANDOM DEMAND
      DIMENSION T(1000),X(1000),IX(1000)
      COMMON IRANDOM,RANDOM
      IRANDOM=3574655
      DO 1 I=1,1000
      CALL NORMAL(16.0,4.0,X(I))
      IX(I)=X(I)
    1 X(I)=IX(I)
      DO 4 I=12,1000
      SUM=0.0
      M=I-11
      DO 2 J=1,M
      TEMP1=2.2*(.8)**(J-1)*X(I-10-J)
    2 SUM=SUM+TEMP1
      TEMP1=SUM
      SUM=0.0
      DO 3 J=1,11
      TEMP2=X(I-J+1)
    3 SUM=SUM+TEMP2
      TEMP2=SUM
    4 T(I)=TEMP1-TEMP2
      PRINT 5
    5 FORMAT(24HRANDOM NUMBERS GENERATED)
      PRINT 6,(X(I),I=1,1000)
    6 FORMAT(6X,5F10.6)
      PRINT 7
    7 FORMAT(17HINVENTORY ON HAND)
      PRINT 8,(T(I),I=12,1000)
    8 FORMAT(6X,5F10.4)
      END
      SUBROUTINE RANVAR
      COMMON IRANDOM,RANDOM
      CON(K1=67108864)
      IRANDOM=IRANDOM*3125
      IF(IRANDOM-67108863)2,2,1
    1 ENQ(0),LDA(IRANDOM),DVI(K1),STQ(IRANDOM)
    2 Y=IRANDOM
      RANDOM=Y/67108864.0
      RETURN
      END
      SUBROUTINE NORMAL(A,B,C)
      COMMON IRANDOM,RANDOM
      SUM=0.0
      DO 1 I=1,12
      CALL RANVAR
    1 SUM=SUM+RANDOM
      X=SUM-6.0
      C=X*B+A
      RETURN
      END
```

```
      PROGRAM POISSON
C THIS PROGRAM SIMULATES BASIC MODEL WITH RANDOM DEMAND
      DIMENSION T(1000),X(1000),IX(1000)
      COMMON IRANDOM,RANDOM
      IRANDOM=3574655
      DO 1 I=1,1000
      CALL NORMAL(100.0,10.0,X(I))
      IX(I)=X(I)
    1 X(I)=IX(I)
      DO 4 I=6,1000
      SUM=0.0
      M=I-5
      DO 2 J=1,M
      TEMP1=(.8)**(J-1)*X(I-4-J)
    2 SUM=SUM+TEMP1
      TEMP1=SUM
      SUM=0.0
      DO 3 J=1,5
      TEMP2=X(I-J+1)
    3 SUM=SUM+TEMP2
      TEMP2=SUM
    4 T(I)=TEMP1-TEMP2
      PRINT 5
    5 FORMAT(24HRANDOM NUMBERS GENERATED)
      PRINT 6,(X(I),I=1,1000)
    6 FORMAT(6X,5F10.3)
      PRINT 7
    7 FORMAT(17HINVENTORY ON HAND)
      PRINT 8,(T(I),I=6,1000)
    8 FORMAT(6X,5F10.4)
      END
      SUBROUTINE RANVAR
      COMMON IRANDOM,RANDOM
      CON(K1=67108864)
      IRANDOM=IRANDOM*3125
      IF(IRANDOM-67108863)2,2,1
    1 ENQ(0),LDA(IRANDOM),DVI(K1),STQ(IRANDOM)
    2 Y=IRANDOM
      RANDOM=Y/67108864.0
      RETURN
      END
      SUBROUTINE NORMAL(A,B,C)
      COMMON IRANDOM,RANDOM
      SUM=0.0
      DO 1 I=1,12
      CALL RANVAR
    1 SUM=SUM+RANDOM
      X=SUM-6.0
      C=X*B+A
      RETURN
      END
```

```
      PROGRAM POISSON
C THIS PROGRAM SIMULATES BASIC MODEL WITH RANDOM DEMAND
      DIMENSION T(1000),X(1000),IX(1000)
      COMMON IRANDOM.RANDOM
      IRANDOM=3574655
      DO 1 I=1,1000
      Z=I
      W=16.0+Z
      U=SQRTF(W)
      CALL NORMAL(W,U,X(I))
      IX(I)=X(I)
    1 X(I)=IX(I)
      DO 4 I=6,1000
      SUM=0.0
      M=I-5
      DO 2 J=1,M
      TEMP1=(.8)**(J-1)*X(I-4-J)
    2 SUM=SUM+TEMP1
      TEMP1=SUM
      SUM=0.0
      DO 3 J=1,5
      TEMP2=X(I-J+1)
    3 SUM=SUM+TEMP2
      TEMP2=SUM
    4 T(I)=TEMP1-TEMP2
      PRINT 5
    5 FORMAT(24HRANDOM NUMBERS GENERATED)
      PRINT 6,(X(I),I=1,1000)
    6 FORMAT(6X,5F10.3)
      PRINT 7
    7 FORMAT(17HINVENTORY ON HAND)
      PRINT 8,(T(I),I=6,1000)
    8 FORMAT(6X,5F10.3)
      END
      SUBROUTINE RANVAR
      COMMON IRANDOM.RANDOM
      CON(K1=67108864)
      IRANDOM=IRANDOM*3125
      IF(IRANDOM-67108863)2,2,1
    1 ENQ(0),LDA(IRANDOM),DVI(K1),STQ(IRANDOM)
    2 Y=IRANDOM
      RANDOM=Y/67108864.0
      RETURN
      END
      SUBROUTINE NORMAL(A,B,C)
      COMMON IRANDOM.RANDOM
      SUM=0.0
      DO 1 I=1,12
      CALL RANVAR
    1 SUM=SUM+RANDOM
      X=SUM-6.0
      C=X*B+A
      RETURN
      END
```

APPENDIX III-A

RANDOM NUMBERS GENERATED

19.000000	10.000000	15.000000	17.000000	8.000000
9.000000	19.000000	15.000000	12.000000	19.000000
20.000000	18.000000	18.000000	20.000000	14.000000
10.000000	20.000000	13.000000	14.000000	12.000000
11.000000	22.000000	12.000000	13.000000	16.000000
10.000000	15.000000	13.000000	23.000000	22.000000
17.000000	8.000000	11.000000	12.000000	18.000000
17.000000	15.000000	18.000000	21.000000	18.000000
12.000000	16.000000	9.000000	13.000000	20.000000
16.000000	11.000000	15.000000	7.000000	15.000000
25.000000	18.000000	19.000000	13.000000	19.000000
13.000000	12.000000	15.000000	22.000000	18.000000
20.000000	11.000000	24.000000	13.000000	14.000000
18.000000	12.000000	14.000000	13.000000	15.000000
22.000000	11.000000	16.000000	11.000000	19.000000
12.000000	18.000000	22.000000	11.000000	16.000000
19.000000	12.000000	13.000000	18.000000	11.000000
12.000000	12.000000	10.000000	21.000000	14.000000
11.000000	19.000000	20.000000	23.000000	13.000000
15.000000	13.000000	6.000000	16.000000	24.000000
23.000000	15.000000	12.000000	24.000000	13.000000
18.000000	19.000000	20.000000	12.000000	16.000000
21.000000	11.000000	14.000000	15.000000	16.000000
10.000000	15.000000	9.000000	16.000000	16.000000
13.000000	8.000000	11.000000	14.000000	16.000000
9.000000	16.000000	6.000000	13.000000	9.000000
22.000000	23.000000	17.000000	13.000000	16.000000
14.000000	13.000000	16.000000	13.000000	13.000000
14.000000	12.000000	17.000000	10.000000	17.000000
22.000000	17.000000	11.000000	15.000000	17.000000
17.000000	23.000000	9.000000	17.000000	20.000000
10.000000	12.000000	19.000000	16.000000	18.000000
20.000000	15.000000	17.000000	19.000000	11.000000
19.000000	18.000000	13.000000	21.000000	21.000000
17.000000	7.000000	12.000000	15.000000	18.000000
19.000000	13.000000	19.000000	15.000000	18.000000
19.000000	16.000000	12.000000	14.000000	9.000000
10.000000	19.000000	18.000000	24.000000	12.000000
15.000000	15.000000	7.000000	11.000000	15.000000
8.000000	17.000000	15.000000	18.000000	21.000000
9.000000	12.000000	12.000000	18.000000	18.000000
18.000000	10.000000	12.000000	12.000000	19.000000
12.000000	9.000000	19.000000	18.000000	17.000000
19.000000	9.000000	9.000000	12.000000	20.000000
15.000000	16.000000	19.000000	21.000000	14.000000
15.000000	19.000000	9.000000	19.000000	15.000000
16.000000	10.000000	17.000000	15.000000	20.000000
18.000000	12.000000	15.000000	7.000000	10.000000
15.000000	19.000000	18.000000	13.000000	11.000000
15.000000	16.000000	12.000000	9.000000	11.000000
19.000000	18.000000	19.000000	14.000000	14.000000
12.000000	21.000000	14.000000	20.000000	20.000000
12.000000	13.000000	16.000000	16.000000	17.000000
12.000000	14.000000	23.000000	18.000000	19.000000
11.000000	14.000000	10.000000	13.000000	7.000000
10.000000	19.000000	15.000000	13.000000	19.000000
17.000000	16.000000	24.000000	17.000000	15.000000
16.000000	18.000000	16.000000	20.000000	8.000000
23.000000	16.000000	14.000000	15.000000	18.000000
19.000000	13.000000	13.000000	17.000000	17.000000
15.000000	18.000000	12.000000	10.000000	17.000000
14.000000	17.000000	12.000000	18.000000	13.000000
12.000000	14.000000	21.000000	20.000000	15.000000
14.000000	23.000000	13.000000	13.000000	16.000000
19.000000	16.000000	17.000000	16.000000	20.000000
19.000000	6.000000	15.000000	13.000000	20.000000
19.000000	20.000000	13.000000	16.000000	11.000000
15.000000	12.000000	22.000000	22.000000	15.000000
14.000000	15.000000	25.000000	11.000000	17.000000

14.000000	15.000000	15.000000	14.000000	12.000000
13.000000	6.000000	10.000000	16.000000	16.000000
14.000000	13.000000	14.000000	24.000000	20.000000
17.000000	18.000000	11.000000	15.000000	14.000000
21.000000	17.000000	13.000000	14.000000	13.000000
12.000000	19.000000	21.000000	12.000000	15.000000
16.000000	21.000000	20.000000	10.000000	13.000000
18.000000	22.000000	11.000000	12.000000	17.000000
10.000000	17.000000	16.000000	23.000000	20.000000
11.000000	10.000000	13.000000	16.000000	13.000000
20.000000	16.000000	21.000000	16.000000	20.000000
13.000000	15.000000	20.000000	16.000000	13.000000
11.000000	12.000000	14.000000	17.000000	9.000000
19.000000	12.000000	9.000000	16.000000	23.000000
16.000000	15.000000	20.000000	16.000000	11.000000
16.000000	17.000000	9.000000	13.000000	17.000000
16.000000	20.000000	17.000000	17.000000	23.000000
13.000000	16.000000	20.000000	16.000000	10.000000
10.000000	20.000000	14.000000	15.000000	17.000000
12.000000	12.000000	21.000000	24.000000	17.000000
24.000000	13.000000	16.000000	14.000000	12.000000
5.000000	16.000000	14.000000	13.000000	13.000000
12.000000	12.000000	19.000000	14.000000	12.000000
15.000000	9.000000	16.000000	21.000000	14.000000
20.000000	10.000000	21.000000	12.000000	15.000000
15.000000	17.000000	4.000000	7.000000	14.000000
17.000000	16.000000	19.000000	17.000000	9.000000
17.000000	16.000000	12.000000	13.000000	15.000000
16.000000	10.000000	17.000000	16.000000	17.000000
17.000000	15.000000	20.000000	18.000000	17.000000
22.000000	21.000000	23.000000	16.000000	16.000000
7.000000	8.000000	14.000000	18.000000	8.000000
13.000000	16.000000	19.000000	11.000000	17.000000
15.000000	16.000000	15.000000	14.000000	13.000000
15.000000	24.000000	22.000000	14.000000	18.000000
11.000000	14.000000	11.000000	16.000000	17.000000
17.000000	8.000000	21.000000	15.000000	17.000000
8.000000	21.000000	6.000000	19.000000	14.000000
8.000000	19.000000	17.000000	13.000000	22.000000
14.000000	18.000000	15.000000	15.000000	14.000000
11.000000	16.000000	14.000000	21.000000	15.000000
11.000000	13.000000	11.000000	18.000000	18.000000
13.000000	15.000000	24.000000	13.000000	17.000000
22.000000	17.000000	10.000000	18.000000	13.000000
25.000000	20.000000	19.000000	14.000000	8.000000
15.000000	17.000000	18.000000	18.000000	13.000000
19.000000	16.000000	15.000000	13.000000	10.000000
19.000000	15.000000	14.000000	20.000000	22.000000
13.000000	20.000000	15.000000	16.000000	18.000000
16.000000	21.000000	16.000000	21.000000	18.000000
21.000000	17.000000	19.000000	17.000000	23.000000
14.000000	15.000000	11.000000	14.000000	13.000000
21.000000	21.000000	20.000000	10.000000	18.000000
16.000000	11.000000	12.000000	21.000000	12.000000
12.000000	21.000000	19.000000	15.000000	14.000000
17.000000	18.000000	16.000000	15.000000	15.000000
7.000000	11.000000	12.000000	12.000000	15.000000
11.000000	20.000000	17.000000	12.000000	26.000000
12.000000	20.000000	15.000000	19.000000	18.000000
13.000000	12.000000	21.000000	13.000000	17.000000
15.000000	15.000000	10.000000	17.000000	23.000000
12.000000	10.000000	14.000000	9.000000	15.000000
10.000000	17.000000	13.000000	15.000000	21.000000
19.000000	14.000000	15.000000	14.000000	20.000000
16.000000	18.000000	10.000000	15.000000	12.000000
18.000000	17.000000	23.000000	13.000000	15.000000
12.000000	10.000000	19.000000	12.000000	8.000000
8.000000	15.000000	23.000000	12.000000	7.000000
16.000000	12.000000	17.000000	15.000000	13.000000
17.000000	14.000000	18.000000	20.000000	17.000000
20.000000	19.000000	15.000000	12.000000	18.000000
22.000000	20.000000	16.000000	17.000000	9.000000

13.000000	20.000000	15.000000	26.000000	11.000000
15.000000	19.000000	11.000000	22.000000	8.000000
18.000000	16.000000	17.000000	14.000000	13.000000
17.000000	13.000000	20.000000	12.000000	8.000000
17.000000	13.000000	20.000000	19.000000	18.000000
8.000000	10.000000	16.000000	11.000000	15.000000
18.000000	11.000000	16.000000	18.000000	16.000000
11.000000	13.000000	20.000000	19.000000	14.000000
15.000000	15.000000	12.000000	16.000000	13.000000
19.000000	15.000000	11.000000	19.000000	22.000000
15.000000	15.000000	21.000000	15.000000	12.000000
14.000000	19.000000	10.000000	12.000000	11.000000
16.000000	13.000000	24.000000	21.000000	10.000000
15.000000	18.000000	16.000000	15.000000	15.000000
12.000000	17.000000	20.000000	16.000000	8.000000
17.000000	16.000000	13.000000	21.000000	9.000000
24.000000	21.000000	11.000000	26.000000	16.000000
15.000000	18.000000	17.000000	10.000000	14.000000
14.000000	10.000000	17.000000	20.000000	14.000000
15.000000	18.000000	13.000000	16.000000	13.000000
3.000000	12.000000	14.000000	16.000000	15.000000
19.000000	13.000000	18.000000	10.000000	10.000000
20.000000	11.000000	16.000000	19.000000	20.000000
17.000000	21.000000	17.000000	16.000000	16.000000
13.000000	24.000000	17.000000	21.000000	16.000000
18.000000	14.000000	21.000000	14.000000	22.000000
16.000000	9.000000	14.000000	23.000000	9.000000
19.000000	13.000000	12.000000	19.000000	14.000000
15.000000	12.000000	16.000000	17.000000	23.000000
18.000000	10.000000	11.000000	22.000000	21.000000
12.000000	18.000000	11.000000	15.000000	14.000000
15.000000	11.000000	13.000000	14.000000	19.000000
17.000000	16.000000	20.000000	14.000000	13.000000
15.000000	17.000000	10.000000	23.000000	11.000000
10.000000	11.000000	15.000000	12.000000	17.000000
15.000000	13.000000	23.000000	13.000000	26.000000
22.000000	19.000000	16.000000	21.000000	8.000000
17.000000	15.000000	22.000000	12.000000	10.000000
16.000000	17.000000	18.000000	16.000000	18.000000
19.000000	22.000000	14.000000	9.000000	13.000000
19.000000	15.000000	9.000000	16.000000	10.000000
18.000000	19.000000	16.000000	22.000000	14.000000
17.000000	16.000000	12.000000	14.000000	17.000000
20.000000	20.000000	13.000000	17.000000	20.000000
12.000000	17.000000	10.000000	6.000000	14.000000
18.000000	16.000000	19.000000	14.000000	15.000000
19.000000	15.000000	14.000000	10.000000	18.000000
16.000000	8.000000	18.000000	14.000000	6.000000
14.000000	19.000000	14.000000	19.000000	18.000000
20.000000	19.000000	15.000000	14.000000	10.000000
20.000000	15.000000	16.000000	22.000000	14.000000
12.000000	18.000000	10.000000	11.000000	11.000000
21.000000	13.000000	14.000000	18.000000	20.000000
18.000000	15.000000	16.000000	16.000000	13.000000
18.000000	20.000000	9.000000	15.000000	20.000000
17.000000	14.000000	15.000000	14.000000	13.000000
8.000000	18.000000	20.000000	17.000000	10.000000
21.000000	14.000000	14.000000	4.000000	16.000000
19.000000	16.000000	7.000000	14.000000	17.000000

APPENDIX III-B

RANDOM NUMBERS GENERATED

109.000	86.000	97.000	104.000	82.000
82.000	108.000	98.000	90.000	107.000
111.000	107.000	105.000	111.000	95.000
86.000	111.000	94.000	95.000	90.000
88.000	116.000	90.000	94.000	100.000
86.000	97.000	93.000	119.000	115.000
104.000	81.000	88.000	92.000	107.000
104.000	97.000	107.000	114.000	106.000
91.000	100.000	82.000	92.000	111.000
100.000	89.000	98.000	79.000	98.000
122.000	106.000	109.000	94.000	109.000
94.000	91.000	97.000	116.000	105.000
110.000	87.000	121.000	94.000	96.000
105.000	91.000	97.000	93.000	99.000
115.000	87.000	102.000	89.000	108.000
90.000	106.000	116.000	89.000	102.000
108.000	90.000	94.000	105.000	89.000
90.000	92.000	86.000	112.000	96.000
89.000	109.000	110.000	117.000	93.000
99.000	94.000	75.000	101.000	122.000
118.000	98.000	90.000	120.000	93.000
106.000	109.000	111.000	91.000	100.000
113.000	88.000	96.000	99.000	101.000
86.000	99.000	83.000	100.000	100.000
93.000	81.000	87.000	96.000	100.000
84.000	100.000	77.000	93.000	82.000
115.000	119.000	103.000	94.000	101.000
96.000	95.000	101.000	94.000	94.000
96.000	91.000	102.000	87.000	104.000
115.000	103.000	89.000	99.000	103.000
103.000	117.000	84.000	104.000	110.000
85.000	90.000	109.000	101.000	107.000
111.000	97.000	104.000	108.000	88.000
108.000	106.000	94.000	114.000	112.000
104.000	79.000	91.000	99.000	106.000
108.000	92.000	109.000	99.000	105.000
109.000	100.000	91.000	96.000	84.000
85.000	108.000	106.000	120.000	91.000
98.000	99.000	78.000	89.000	98.000
80.000	103.000	99.000	106.000	114.000
84.000	91.000	90.000	106.000	106.000
105.000	85.000	91.000	90.000	109.000
91.000	83.000	109.000	105.000	103.000
108.000	84.000	82.000	90.000	110.000
99.000	100.000	109.000	114.000	96.000
99.000	107.000	84.000	107.000	98.000
101.000	87.000	104.000	98.000	110.000
106.000	90.000	98.000	79.000	87.000
98.000	108.000	107.000	94.000	88.000
98.000	101.000	92.000	84.000	87.000
108.000	106.000	109.000	97.000	95.000
90.000	114.000	96.000	111.000	111.000

92.000	94.000	101.000	102.000	104.000
91.000	95.000	118.000	105.000	108.000
88.000	95.000	85.000	93.000	78.000
86.000	107.000	97.000	93.000	108.000
104.000	100.000	120.000	104.000	98.000
101.000	106.000	100.000	111.000	80.000
119.000	100.000	96.000	97.000	106.000
109.000	94.000	92.000	104.000	103.000
99.000	105.000	91.000	85.000	102.000
96.000	104.000	91.000	107.000	94.000
90.000	96.000	114.000	110.000	97.000
95.000	117.000	92.000	92.000	101.000
108.000	102.000	102.000	101.000	110.000
108.000	76.000	99.000	94.000	111.000
109.000	110.000	93.000	100.000	89.000
98.000	90.000	116.000	115.000	99.000
96.000	98.000	122.000	87.000	104.000
97.000	99.000	98.000	96.000	91.000
93.000	75.000	86.000	100.000	101.000
95.000	94.000	96.000	120.000	111.000
104.000	107.000	88.000	99.000	97.000
113.000	102.000	93.000	97.000	94.000
90.000	109.000	114.000	90.000	99.000
100.000	113.000	111.000	86.000	93.000
105.000	117.000	88.000	92.000	102.000
87.000	104.000	101.000	118.000	111.000
89.000	85.000	93.000	100.000	92.000
110.000	101.000	112.000	100.000	111.000
93.000	99.000	111.000	101.000	93.000
89.000	91.000	96.000	103.000	83.000
108.000	91.000	84.000	100.000	119.000
100.000	99.000	110.000	102.000	89.000
101.000	104.000	83.000	92.000	102.000
100.000	111.000	104.000	104.000	119.000
93.000	100.000	110.000	101.000	86.000
85.000	112.000	95.000	97.000	104.000
92.000	92.000	114.000	121.000	102.000
121.000	94.000	101.000	96.000	91.000
74.000	100.000	95.000	94.000	92.000
91.000	91.000	108.000	95.000	90.000
98.000	84.000	101.000	112.000	97.000
111.000	86.000	112.000	90.000	99.000
99.000	104.000	71.000	78.000	95.000
103.000	101.000	109.000	104.000	84.000
102.000	100.000	92.000	93.000	98.000
101.000	86.000	103.000	100.000	103.000
104.000	99.000	111.000	105.000	103.000
117.000	113.000	118.000	101.000	101.000
78.000	80.000	95.000	107.000	80.000
92.000	102.000	108.000	89.000	104.000
99.000	100.000	99.000	95.000	94.000
98.000	121.000	116.000	95.000	106.000
89.000	96.000	88.000	100.000	104.000
104.000	81.000	114.000	99.000	102.000

81.000	113.000	77.000	108.000	96.000
81.000	108.000	103.000	93.000	115.000
97.000	107.000	99.000	98.000	95.000
88.000	101.000	96.000	114.000	98.000
88.000	92.000	88.000	105.000	106.000
93.000	97.000	121.000	93.000	103.000
115.000	104.000	85.000	107.000	94.000
124.000	112.000	109.000	97.000	82.000
99.000	104.000	105.000	107.000	93.000
108.000	101.000	99.000	94.000	86.000
107.000	97.000	96.000	111.000	116.000
93.000	110.000	99.000	100.000	105.000
100.000	114.000	101.000	113.000	105.000
113.000	102.000	107.000	102.000	118.000
95.000	98.000	89.000	96.000	93.000
114.000	113.000	111.000	87.000	106.000
100.000	88.000	90.000	114.000	90.000
92.000	114.000	107.000	98.000	97.000
103.000	106.000	102.000	98.000	97.000
79.000	89.000	91.000	90.000	98.000
88.000	110.000	103.000	91.000	125.000
90.000	110.000	99.000	107.000	106.000
92.000	91.000	112.000	93.000	102.000
98.000	98.000	85.000	102.000	119.000
91.000	87.000	95.000	84.000	99.000
85.000	103.000	93.000	99.000	112.000
107.000	95.000	98.000	95.000	112.000
101.000	105.000	87.000	98.000	91.000
106.000	104.000	118.000	94.000	99.000
90.000	85.000	109.000	90.000	80.000
80.000	98.000	118.000	90.000	77.000
100.000	90.000	103.000	98.000	94.000
103.000	95.000	105.000	110.000	104.000
111.000	109.000	99.000	90.000	106.000
115.000	110.000	101.000	103.000	83.000
93.000	110.000	99.000	127.000	88.000
98.000	108.000	88.000	115.000	81.000
105.000	100.000	103.000	96.000	94.000
103.000	92.000	112.000	91.000	82.000
103.000	94.000	110.000	107.000	105.000
81.000	87.000	102.000	87.000	99.000
105.000	88.000	100.000	107.000	101.000
89.000	94.000	111.000	108.000	96.000
99.000	99.000	92.000	102.000	93.000
108.000	97.000	89.000	108.000	115.000
99.000	99.000	113.000	99.000	92.000
95.000	107.000	85.000	90.000	89.000
101.000	93.000	120.000	114.000	85.000
98.000	105.000	101.000	98.000	98.000
91.000	104.000	110.000	101.000	82.000
104.000	101.000	94.000	113.000	84.000
121.000	114.000	89.000	126.000	100.000
99.000	105.000	102.000	87.000	95.000
96.000	86.000	103.000	111.000	96.000

98.000	107.000	94.000	101.000	93.000
69.000	90.000	95.000	101.000	98.000
108.000	94.000	105.000	86.000	85.000
111.000	88.000	101.000	107.000	112.000
102.000	114.000	103.000	101.000	100.000
94.000	120.000	104.000	112.000	101.000
105.000	96.000	113.000	96.000	115.000
100.000	84.000	97.000	117.000	83.000
108.000	94.000	90.000	108.000	97.000
99.000	91.000	101.000	103.000	119.000
106.000	87.000	89.000	116.000	113.000
90.000	107.000	89.000	99.000	95.000
99.000	88.000	93.000	97.000	108.000
102.000	100.000	111.000	96.000	92.000
99.000	102.000	85.000	118.000	87.000
85.000	88.000	98.000	91.000	104.000
98.000	94.000	119.000	94.000	125.000
115.000	107.000	100.000	114.000	81.000
104.000	98.000	115.000	91.000	86.000
101.000	102.000	106.000	101.000	105.000
107.000	115.000	96.000	84.000	92.000
108.000	99.000	84.000	100.000	86.000
107.000	109.000	102.000	117.000	97.000
103.000	101.000	92.000	96.000	103.000
110.000	110.000	93.000	103.000	111.000
92.000	104.000	87.000	77.000	97.000
106.000	100.000	108.000	95.000	98.000
108.000	99.000	96.000	85.000	105.000
101.000	80.000	107.000	96.000	76.000
96.000	109.000	95.000	107.000	105.000
110.000	109.000	97.000	97.000	87.000
110.000	97.000	100.000	116.000	95.000
91.000	105.000	86.000	89.000	89.000
113.000	94.000	95.000	105.000	111.000
107.000	99.000	102.000	102.000	92.000
105.000	110.000	84.000	97.000	110.000
103.000	97.000	99.000	96.000	94.000
81.000	106.000	110.000	103.000	86.000
114.000	96.000	95.000	71.000	101.000
109.000	100.000	77.000	95.000	104.000

APPENDIX III-C

RANDOM NUMBERS GENERATED

20.000	12.000	17.000	22.000	12.000
13.000	27.000	23.000	20.000	29.000
33.000	31.000	32.000	36.000	28.000
24.000	39.000	30.000	32.000	30.000
30.000	48.000	33.000	36.000	41.000
33.000	41.000	39.000	58.000	56.000
49.000	35.000	40.000	44.000	56.000
55.000	51.000	59.000	65.000	61.000
50.000	58.000	45.000	54.000	69.000
62.000	54.000	62.000	48.000	65.000
85.000	73.000	76.000	65.000	78.000
67.000	66.000	72.000	89.000	80.000
86.000	67.000	98.000	75.000	77.000
86.000	75.000	81.000	78.000	85.000
101.000	76.000	91.000	80.000	99.000
83.000	99.000	110.000	84.000	97.000
105.000	88.000	93.000	105.000	90.000
92.000	95.000	90.000	118.000	101.000
96.000	118.000	119.000	128.000	104.000
111.000	107.000	87.000	116.000	139.000
136.000	116.000	109.000	142.000	113.000
129.000	135.000	136.000	115.000	126.000
142.000	114.000	125.000	129.000	132.000
116.000	132.000	114.000	135.000	137.000
129.000	116.000	124.000	135.000	141.000
124.000	143.000	116.000	136.000	125.000
165.000	171.000	153.000	143.000	152.000
147.000	145.000	155.000	148.000	148.000
153.000	147.000	162.000	143.000	166.000
181.000	167.000	151.000	164.000	170.000
171.000	190.000	148.000	176.000	184.000
152.000	160.000	186.000	177.000	185.000
191.000	174.000	185.000	191.000	165.000
193.000	192.000	176.000	204.000	203.000
193.000	159.000	177.000	189.000	200.000
203.000	182.000	206.000	194.000	203.000
210.000	199.000	186.000	195.000	179.000
180.000	214.000	213.000	234.000	193.000
204.000	207.000	178.000	195.000	209.000
183.000	217.000	213.000	225.000	237.000
194.000	205.000	204.000	230.000	230.000
229.000	201.000	211.000	210.000	240.000
213.000	204.000	242.000	238.000	236.000
244.000	209.000	207.000	220.000	251.000
236.000	239.000	253.000	261.000	235.000
240.000	254.000	220.000	256.000	243.000
250.000	228.000	256.000	247.000	267.000
262.000	236.000	250.000	222.000	235.000
253.000	271.000	271.000	250.000	242.000
258.000	265.000	251.000	240.000	246.000
280.000	276.000	284.000	265.000	264.000
257.000	296.000	267.000	293.000	295.000

263.000	268.000	282.000	284.000	289.000
268.000	274.000	315.000	293.000	300.000
266.000	280.000	264.000	279.000	254.000
269.000	306.000	290.000	283.000	310.000
305.000	298.000	334.000	307.000	298.000
304.000	314.000	304.000	325.000	271.000
341.000	308.000	302.000	306.000	322.000
329.000	303.000	301.000	323.000	321.000
316.000	328.000	303.000	294.000	326.000
315.000	331.000	309.000	337.000	316.000
310.000	322.000	355.000	348.000	326.000
323.000	365.000	320.000	321.000	339.000
353.000	341.000	344.000	343.000	360.000
357.000	298.000	343.000	334.000	367.000
364.000	366.000	336.000	351.000	332.000
349.000	334.000	385.000	383.000	354.000
349.000	356.000	401.000	336.000	369.000
356.000	362.000	380.000	358.000	349.000
354.000	321.000	342.000	371.000	373.000
362.000	362.000	366.000	414.000	398.000
384.000	391.000	356.000	378.000	376.000
407.000	388.000	371.000	379.000	375.000
368.000	406.000	418.000	371.000	390.000
393.000	420.000	416.000	368.000	383.000
408.000	432.000	376.000	384.000	406.000
376.000	411.000	406.000	441.000	429.000
386.000	379.000	395.000	410.000	396.000
434.000	415.000	440.000	416.000	438.000
404.000	416.000	442.000	422.000	408.000
400.000	404.000	416.000	431.000	391.000
443.000	409.000	396.000	430.000	470.000
433.000	432.000	456.000	439.000	414.000
440.000	447.000	405.000	424.000	446.000
443.000	468.000	453.000	455.000	488.000
433.000	449.000	471.000	453.000	422.000
421.000	479.000	445.000	450.000	465.000
440.000	441.000	490.000	505.000	467.000
507.000	451.000	467.000	457.000	448.000
412.000	468.000	458.000	457.000	455.000
453.000	455.000	492.000	466.000	456.000
473.000	443.000	482.000	508.000	474.000
506.000	453.000	511.000	464.000	485.000
484.000	497.000	425.000	443.000	481.000
500.000	495.000	514.000	505.000	462.000
503.000	498.000	481.000	484.000	497.000
505.000	472.000	510.000	506.000	514.000
517.000	506.000	535.000	523.000	519.000
551.000	543.000	555.000	518.000	519.000
469.000	473.000	508.000	536.000	475.000
505.000	527.000	543.000	500.000	536.000
525.000	529.000	527.000	520.000	518.000
528.000	583.000	571.000	523.000	551.000
512.000	529.000	511.000	540.000	551.000
553.000	499.000	577.000	544.000	552.000

503.000	578.000	495.000	571.000	542.000
509.000	573.000	563.000	538.000	592.000
550.000	574.000	557.000	556.000	549.000
535.000	567.000	554.000	600.000	562.000
540.000	550.000	540.000	584.000	587.000
556.000	567.000	625.000	559.000	584.000
615.000	588.000	545.000	596.000	568.000
641.000	612.000	605.000	578.000	543.000
584.000	599.000	601.000	607.000	575.000
613.000	596.000	591.000	580.000	562.000
615.000	592.000	591.000	627.000	641.000
585.000	627.000	602.000	606.000	618.000
608.000	644.000	613.000	642.000	624.000
644.000	619.000	633.000	622.000	662.000
606.000	613.000	593.000	611.000	603.000
658.000	656.000	653.000	592.000	642.000
629.000	599.000	604.000	666.000	606.000
611.000	670.000	653.000	630.000	628.000
644.000	653.000	644.000	637.000	634.000
590.000	617.000	623.000	620.000	641.000
617.000	675.000	659.000	628.000	715.000
628.000	680.000	652.000	674.000	673.000
638.000	636.000	691.000	642.000	668.000
658.000	658.000	625.000	672.000	716.000
644.000	635.000	657.000	629.000	668.000
633.000	681.000	656.000	672.000	708.000
697.000	667.000	676.000	667.000	712.000
687.000	697.000	650.000	679.000	663.000
704.000	698.000	737.000	674.000	689.000
666.000	654.000	720.000	671.000	644.000
646.000	694.000	749.000	673.000	642.000
702.000	678.000	713.000	701.000	690.000
715.000	696.000	722.000	739.000	722.000
743.000	739.000	713.000	690.000	732.000
758.000	747.000	720.000	730.000	677.000
705.000	752.000	722.000	798.000	695.000
723.000	750.000	698.000	771.000	681.000
745.000	733.000	742.000	726.000	721.000
747.000	718.000	771.000	716.000	692.000
751.000	727.000	773.000	765.000	759.000
695.000	713.000	755.000	715.000	749.000
767.000	722.000	756.000	774.000	759.000
728.000	743.000	789.000	783.000	751.000
759.000	760.000	743.000	771.000	748.000
789.000	761.000	738.000	793.000	814.000
770.000	770.000	810.000	773.000	753.000
763.000	799.000	739.000	754.000	751.000
786.000	763.000	840.000	824.000	744.000
782.000	803.000	793.000	785.000	787.000
766.000	804.000	823.000	799.000	745.000
808.000	802.000	782.000	837.000	755.000
862.000	843.000	774.000	879.000	806.000
804.000	824.000	816.000	774.000	798.000
802.000	773.000	822.000	848.000	807.000

813.000	838.000	832.000	825.000	801.000
733.000	796.000	812.000	829.000	822.000
852.000	810.000	845.000	791.000	788.000
864.000	800.000	839.000	857.000	871.000
845.000	880.000	849.000	844.000	841.000
826.000	902.000	857.000	882.000	851.000
861.000	836.000	888.000	838.000	895.000
853.000	808.000	845.000	907.000	806.000
882.000	842.000	831.000	884.000	853.000
859.000	836.000	868.000	875.000	923.000
884.000	829.000	837.000	918.000	910.000
842.000	894.000	842.000	874.000	861.000
875.000	842.000	858.000	872.000	905.000
889.000	885.000	918.000	873.000	863.000
886.000	895.000	846.000	944.000	854.000
847.000	859.000	890.000	870.000	908.000
891.000	882.000	956.000	882.000	978.000
948.000	925.000	906.000	947.000	849.000
919.000	902.000	956.000	884.000	869.000
915.000	921.000	933.000	918.000	931.000
939.000	964.000	908.000	872.000	898.000
948.000	921.000	875.000	925.000	883.000
948.000	955.000	936.000	982.000	922.000
944.000	938.000	909.000	925.000	947.000
970.000	970.000	913.000	950.000	977.000
918.000	955.000	905.000	874.000	938.000
967.000	950.000	973.000	936.000	945.000
978.000	950.000	943.000	911.000	973.000
960.000	897.000	981.000	948.000	888.000
952.000	993.000	951.000	989.000	984.000
1000.000	996.000	961.000	961.000	931.000
1005.000	966.000	975.000	1027.000	963.000
949.000	996.000	937.000	945.000	948.000
1022.000	966.000	968.000	1003.000	1023.000
1010.000	985.000	995.000	997.000	967.000
1010.000	1026.000	943.000	987.000	1028.000
1009.000	986.000	997.000	990.000	982.000
942.000	1022.000	1037.000	1016.000	961.000
1052.000	997.000	993.000	919.000	1017.000
1042.000	1014.000	945.000	1001.000	1029.000

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13. ABSTRACT

A [illegible] inventory control model, formulated in terms of [illegible] servomechanism theory, is examined. The behavior of the system is first analyzed by digital computer simulation. Both random and [illegible] inputs are considered. Two double echelon models are then constructed and examined analytically utilizing transform theory. Finally, the properties of forecastors of demand are analyzed for the cases where mean demand is constant and linear. Exponential smoothing techniques are compared with standard estimation procedures.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Servomechanism						
Dynamic Inventory Model						

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